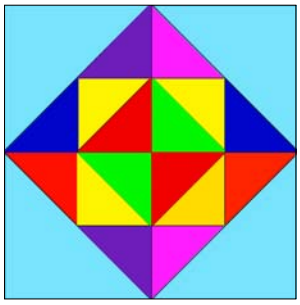
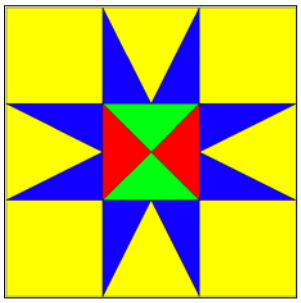
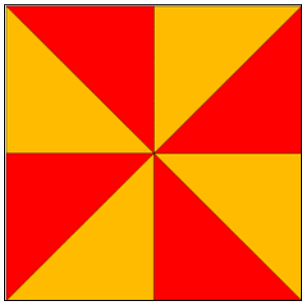


Learning Mathematics in the Primary Grades







Teaching and Learning—Math Division

Madison Metropolitan School District

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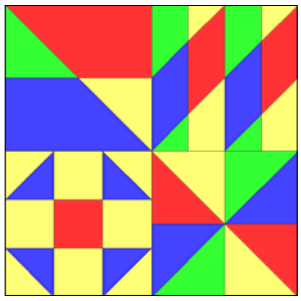
Forward

Many thanks to the learners from across the
Madison Metropolitan School District,
both teachers and students,
who have contributed their ideas
and shared their insights.

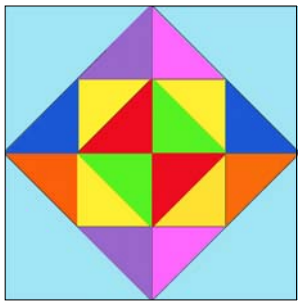


"Our mission is to assure that every student has the knowledge and skills needed for academic achievement and a successful life."

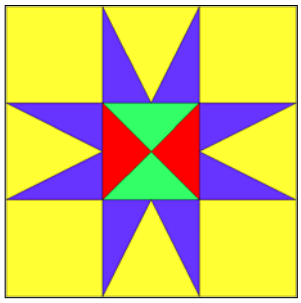
Strategic Plan
Madison Metropolitan School District
2004



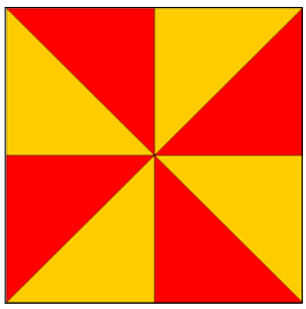
PROBLEM SOLVING



NUMBER WORK



INSPECTING EQUATIONS

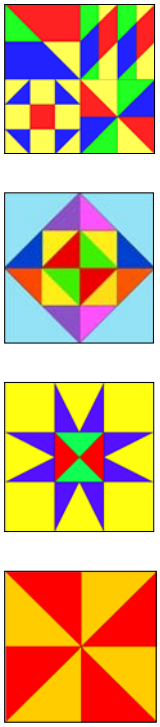


FLUENCY & MAINTENANCE

Chapter 1

Learning Mathematics in the Primary Grades

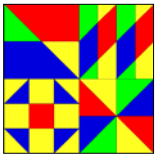
Learning Mathematics in the Primary Grades

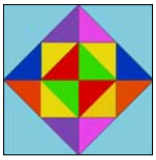


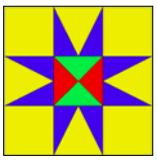
For their first five years, children build informal and intuitive understandings about numbers, shapes, and sizes. To do this, they investigate quantities, shapes and locations by playing with the things in their environment. Parents, older siblings and other adults enrich young children's experiences by teaching them how to count, by playing counting games and by showing them how to flip, slide or turn puzzle pieces to get them to fit. When the environment is organized to foster putting like things together, such as putting away the blocks in one place and the vehicles in another, children learn to compare and classify—two important aspects of mathematical thinking. Young children develop a disposition for mathematics through their early years of interacting with the people and things in their environment.



When children begin kindergarten, they rely on the informal, intuitive mathematical understandings developed in the first five









years of their lives to solve story problems involving joining, separating, grouping, and partitioning. The first problems they meet are story problems that have familiar contexts. The actions in the problems help primary grade students act out or model the situations. They capture the quantities with real objects or counters and then use their counting skills to answer the questions in the problems. Students should solve problems with numbers as well as problems using two- and three-dimensional shapes. They should be encouraged to talk about what they notice during their problem solving experiences.

As students in the primary grades deepen their math knowledge and learn more about conventional mathematical representations and vocabulary, they become more flexible in the ways they work with numbers and shapes. They are more able to apply their knowledge and skills to solve a wider variety of problems with unfamiliar as well as familiar contexts.

It is imperative that students in the early grades feel confident and believe that they are capable and competent mathematics learners.





In kindergarten, first, and second grade, students' math learning experiences:

- ☑ focus on solving problems involving numbers, shapes, measurements and data
- ☑ move from using objects to using numbers and number relationships to model problems and determine solutions
- ☑ foster conversations and asking questions about solution strategies
- ☑ support students' engagement in an orderly progression of high-level mathematical thinking and reasoning experiences



In kindergarten, first, and second grade, students' math learning experiences:

- ☑ begin with the familiar to help make sense of the new
- ☑ build on the knowledge and skills students already possess
- ☑ are based on continuous assessments of progress
- ☑ engage each student in working just on the edge of her/his understanding
- ☑ occur in a variety of group sizes in order to be more effective in meeting specific individual needs



In kindergarten, first, and second grade, students' math learning experiences:

- ☑ encourage students to expect that math makes sense
- ☑ allow students to use personally meaningful strategies to solve problems
- ☑ include time for students to explain their strategies to classmates and teachers
- ☑ offer ways to make paper and pencil records of the thinking steps taken to solve problems
- ☑ provide time and tools for students to determine for themselves the accuracy of their solutions



In kindergarten, first, and second grade, students' math learning experiences:

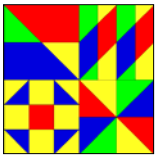
- ☑ develop mathematical ideas as well as practice skills
- ☑ allow time for conceptual development as well as attainment of vocabulary and computational skills
- ☑ support students' efforts to construct relationships between mathematical ideas
- ☑ provide time for each student to build fluency through practice of known concepts and computation



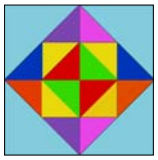
In kindergarten, first, and second grade, students' math learning experiences:

☑ build each student's proficiency in:

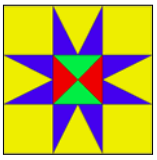
- understanding concepts, operations, and relationships
- computing flexibly, efficiently and accurately
- applying strategies based on conceptual and procedural knowledge in order to solve problems
- explaining solution strategies, reflecting on and justifying the efficacy of each strategy, and making connections to new situations
- engaging willingly in challenging mathematical experiences and feeling capable of being successful.



Problem Solving



Number Work



Inspecting Equations



Fluency & Maintenance

For More Information

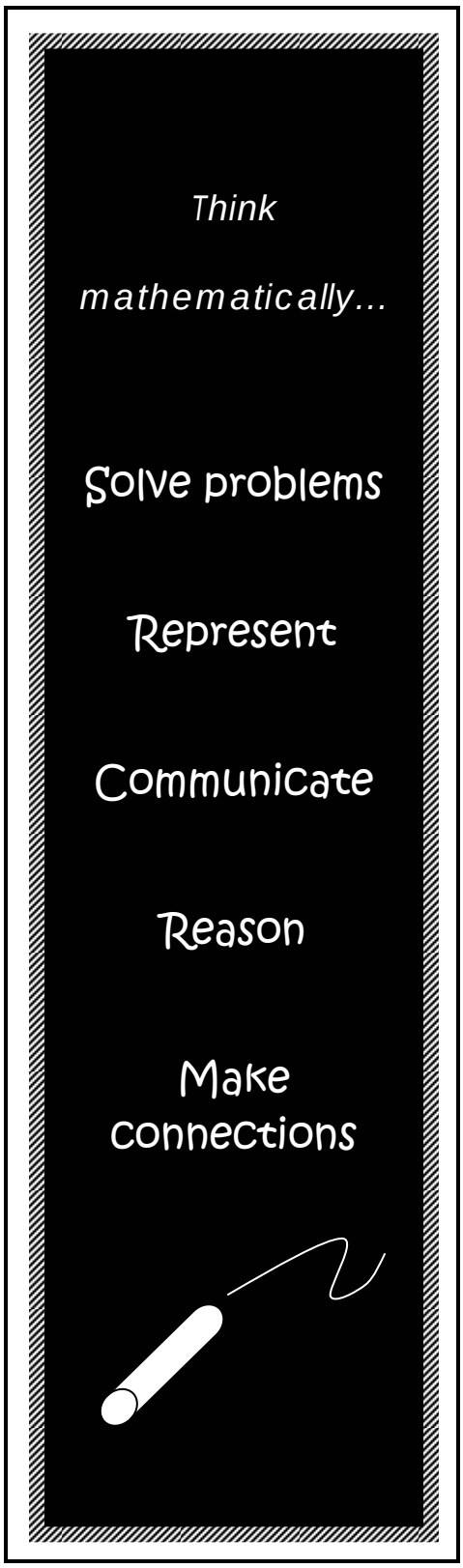
Clarke, B. & Clarke, D. (2004). Mathematics teaching in grades K-2: Painting a picture of challenging, supportive, and effective classrooms. In Rubenstein, R. N. & Bright, G. W. (Eds.). *Perspectives on the Teaching of Mathematics Sixty-sixth Yearbook.* (pp. 67-81). Reston, VA: National Council of Teachers of Mathematics.

Fuson, K. C., Kalchman, M., & Bransford, J. D. (2005). Mathematical understanding: An introduction. *In Donovan, M. S., & Bransford, J. D. (Eds.). How Students Learn Mathematics In the Classroom.* (pp. 215-256). Washington, DC: The National Academies Press.

Kilpatrick, J., Swafford, J., & Findell. (Eds.). (2001). *Adding it up*. Washington, DC: National Academy Press.

Kilpatrick, J., Swafford, J., & Findell. (Eds.). *(2002). Helping students learn mathematics.* Washington, DC: National Academy Press.

Malloy, C. E. (2004). Equity in mathematics education is about access. In Rubenstein, R. N. & Bright, G. W. (Eds.). *Perspectives on the Teaching of Mathematics Sixty-sixth Yearbook.* (pp. 1-14). Reston, VA: National Council of Teachers of Mathematics.



Chapter 2

Math Content and Processes

Mathematics in the K-2 Program:
Math Content and Processes

Math Content

The content of primary mathematics is more than just arithmetic. Current content includes **number and operations, geometry, measurement, data analysis** and beginning experiences with **probability.** Today, kindergarteners, first, and second graders also engage in **algebraic relationship** activities.

Primary math instruction focuses on the learning of both math content and math processes.

The Madison Metropolitan School District K – 5 Grade Level Mathematics Standards organize math concepts and knowledge into four strands of mathematics content: **number, operations and algebraic relationships; geometry; measurement; and data analysis and probability**. Students in kindergarten, first and second grade need to learn the content in all four strands. However, most aspects of primary math experiences, no matter in which content strand, are based on using number concepts. For example, a student filling a given space with pattern blocks will often count the number of each kind of block he used (using number in the geometry strand). To find out how long a ribbon is, a student may place clothespins next to the ribbon to match the length and then count those clothespins (using number in the measurement strand). Analyzing class lunch data involves counting each set – how many have hot lunch, cold lunch, are going home for lunch – before recording the data on a graph. The class may use this graph to count the differences between the sets (using number in data analysis). Experiences with number thread their way through all of the math content strands in the primary grades.

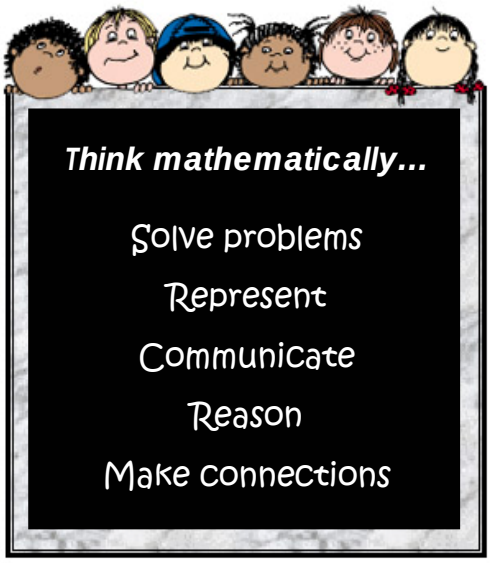
Primary math experiences in all of the content strands are based on using number concepts.

Math Processes

If the math content strands are the nouns of the primary math program, the math processes are the verbs!

Today's students need to be proficient in using the processes of thinking mathematically.

Students' math learning cannot be limited to learning procedures. Today's students must know how to use their math knowledge and skills in flexible as well as efficient ways. They have to engage willingly in ***solving problems*** whether the context is familiar or unfamiliar. They should construct models to ***represent*** their understanding of the problem. They must ***communicate*** their thinking steps and mathematical ideas clearly. They need to ***reason*** about the accuracy of their solutions and convince themselves of the reasonableness of their answers. They must seek patterns and make conjectures about those patterns. They have to expect to ***make connections*** within and across content strands. Today's students need to develop proficiencies in using all five processes in order to think mathematically!



*The math **processes** are:*

- using a variety of strategies to solve problems **(problem solving)**
- using representations (objects, pictures, words and symbols) to organize one's thinking and to record the thinking steps taken to solve a problem **(representation)**
- using the language of mathematics to express and explain mathematical ideas **(communication)**
- making conjectures, identifying examples that show the conjectures to be true or not, and thinking about how and why one understands this to be so **(reasoning and proof)**
- seeing the connections among ideas within mathematics and between mathematics and everyday experiences **(connections)**

Specific content and process expectations at kindergarten, first and second grade can be found in the Madison Metropolitan School District's *MMSD K-5 Grade Level Mathematics Standards.*

For More Information:

Clements, D. H. & Sarama, J. (Eds.). *(*2004). *Engaging young students in mathematics: Standards for early childhood mathematics education.* Malwah, NJ: Lawrence Erlbaum Associates, Publishers.

Kilpatrick, J., Martin, G. W. & Schifter, D. (Eds.). (2003). *A research companion to principles and standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Madison Metropolitan School District. *(2004). MMSD mathematics grade level K-5 standards*. Madison, WI: Retrieved March 16, 2006 from http://www.madison.k12.wi.us/tnl/standards/math/

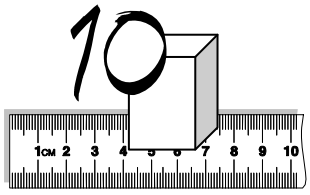
National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: An on-line version retrieved March 16, 2006 from http://standards.nctm.org/

Sawder, J., & Schapelle, B. (Eds.). (2002). *Lessons learned from research.* Reston, VA: National Council of Teachers of Mathematics.

Wisconsin Department of Public Instruction. *Wisconsin model academic standards.* Madison, WI*:* Retrieved March 16, 2006 from http://dpi.wi.gov/standards/matintro.html

Wisconsin Department of Public Instruction. (2005). *Wisconsin knowledge and concepts examination assessment framework.* Madison, WI: Retrieved March 16, 2006 from http://www.dpi.state.wi.us/oea/doc/wkce_math_framework05.doc

Primary Grade Mathematics



Assess

Evaluate

Plan

Teach

Chapter 3

Teaching and Learning Cycle

Role of Teacher

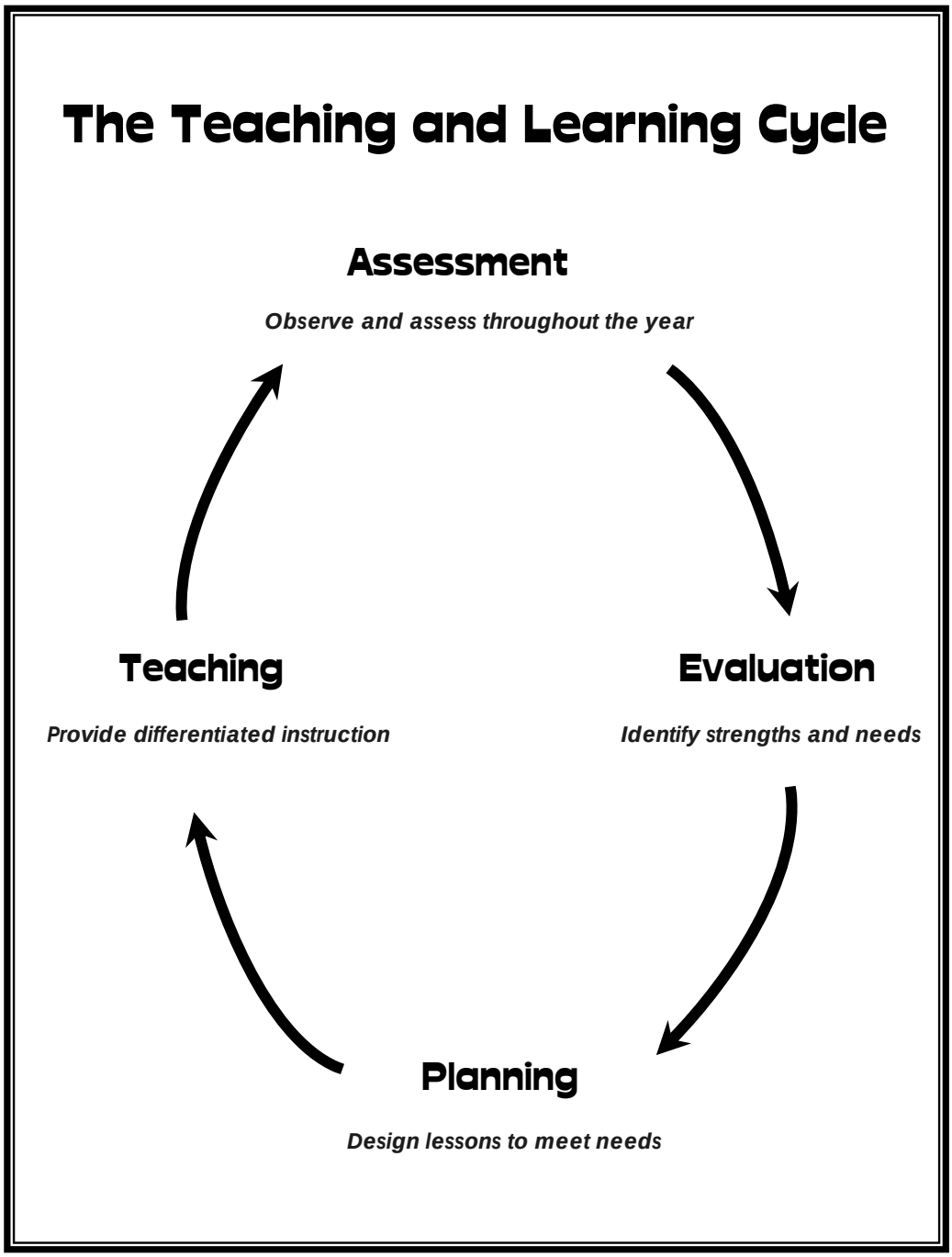
The Teaching Learning Cycle

"The key to teaching students is figuring out what and how they are thinking while the teaching and learning are actually happening. Teaching and learning occur in a social context as a dynamic process rather than as a preconceived one. Lev Vygotsky's work is based on this idea. The basic premise of his theory is that, if we want to study how students learn, to assess their potential to learn, and to improve instruction, we must analyze their performance and their thinking while they are engaged in learning activities. This is what effective teachers do daily."

Vygotsky in the Classroom
Dixon-Krauss

The Teaching and Learning Cycle begins with assessment. Both formal and informal assessments at the beginning of the year are essential to establish the point at which a teacher begins instruction. Daily, ongoing observation and assessment continue throughout the year to identify strengths and needs and ascertain how each child's understandings have changed. Being a skilled observer of children is one of the teacher's most important tasks. Evaluating each child's understandings helps to determine the effectiveness of the instruction and informs future planning. Based on this thoughtful planning, instruction is differentiated to scaffold learning for all students. The cycle continues as the teacher reassesses students, evaluates the assessment results, reflects on lessons taught, and plans and teaches new lessons. The Teaching and Learning Cycle diagram that follows illustrates the recursive nature of assessing, evaluating, planning and teaching.

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Reprinted with permission. Esser, D., Gleason, C., Kolan, T., Lucas, P. & Rohde, J. (2005). *Primary Literacy Notebook.* Madison, WI: Madison Metropolitan School District.

Role of the Teacher

What is the teacher's role during problem solving activities?

Most of math instruction focuses on solving problems. During problem solving activities, the teacher's role is to be a thoughtful mathematician and a careful observer. The teacher begins by posing a problem and observing students' efforts to solve that problem. The teacher supports and guides each student's mathematical growth by posing questions that focus students' attention and encourage reflection.

- *Begin by posing a problem for the students to consider.*
- *Avoid thinking aloud or demonstrating how to solve a problem.*
- *Carefully watch how each student proceeds in finding his or her own solutions to the problem.*
- *Expect students to select the tools they need in order to solve the problems in ways that make sense.*
- *Show confidence in the students' abilities and initiative.*
- *Use thoughtful questions to probe students' thinking and to support and to clarify student explanations.*
- *Engage students in comparing and contrasting solutions to help them expand and extend their understandings.*
- *Use questions to help students build connections between mathematical ideas.*
- *Ask students to reflect on the important mathematical concepts and knowledge they have learned during the problem solving experience.*



What tends to happen if a teacher models his/her thinking before students have a chance to work on solving the problem?

If, during problem-solving activities, a teacher regularly begins by thinking out loud or modeling solution strategies, students begin to wonder if their teacher has a preconceived expectation of the steps or procedures they should use to solve a given problem.

- If students think that there is an expected solution strategy or procedure, they often wait for teachers to prompt or to guide their thinking.
- If students think that there is one way to solve a problem, they focus on trying to remember procedures and information instead of on using mathematical concepts and knowledge in flexible ways.
- If students wait for teachers to acknowledge the accuracy or inaccuracy of their solutions, they lose confidence in their own abilities. They become less engaged in the mathematical discussions and learning opportunities are lost.

Is it ever appropriate to teach a mathematical idea explicitly?

From time to time in math instruction, students need to learn math conventions such as:

- how to write a 5
- what number comes after 109
- what the symbol for addition is
- how to use an empty number line to record the thinking steps for a solution strategy

When teaching mathematical conventions, the teacher uses the same teaching strategies used when introducing new literacy concepts or processes. For example, the teacher models and thinks aloud about a symbol's attributes or the process of constructing a conventional representation. Gradually, the teacher releases responsibility and expects the students to become more and more independent in their use of these mathematical conventions.

What do teachers consider when selecting problems?

To deepen their own mathematical knowledge, teachers always ask themselves, "Why does this work?"

Prior to engaging students in problem solving, the teacher selects problems that will move students forward in building their mathematical proficiencies. The teacher uses her/his own knowledge of the mathematics and her/his understanding of each student's needs when selecting problems. Teachers select problems that will be most effective in learning valuable mathematical ideas and skills. Problems should invite participation by all students.

When selecting **which** problems to pose, the teacher attends to:

- assessments of students' math understandings
- ways students model the problems
- language and representations that students use to share their solution strategies

Using the goals outlined in the Madison Metropolitan School District K – 5 Grade Level Standards, primary math teachers provide a series of problem solving experiences as well as number work experiences to foster each student's growth from a high dependency on concrete models (manipulatives) to a reliance on mental constructs and images.

For more information:

Chappell, M.F., Schielack, J. F. & Zagorski, S. (Eds.). (2004). *Empowering the beginning teacher of mathematics: Elementary school.* Reston, VA: National Council of Teachers of Mathematics.

Dixon-Krauss, Lisbeth. (1996). *Vygotsky in the classroom: Mediated literacy instruction and assessment*. Boston, MA: Allyn and Bacon.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K.C., Wearne, D. & Hanlie, M. (1997). *Making sense: Teaching and learning mathematics with understanding.* Portsmouth, NH: Heinemann.

Ma, L. (1999). *Knowing and teaching elementary mathematics Chapter 5.* Mahwah, NJ: Lawrence Erlbaum Associates, Publishers.

White, D. Y. (2000). Reaching all students mathematically through questioning. In Strutchens, M. E., Johnson, M. L. & Tate, W. F. (Eds.). *Changing the face of mathematics: Perspectives on African-Americans.* (pp. 21-32). Reston, VA: National Council of Teachers of

Teacher Observations

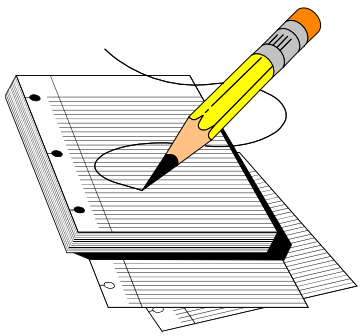
Work Samples

Informal Inventories

Fact Interviews

Problem Solving Interviews

Primary Math Assessments



Chapter 4

Assessment

Assessment

There are many ways for teachers to collect information about each student's progress in achieving mathematical knowledge, proficiencies and fluency. A teacher needs to determine the purposes of the assessments before deciding which assessment information to collect.

Purposes and Ways to Collect Assessment Information

A balance in the kinds of assessments yields the most effective portrait of student progress.

- ☑ To have the most current information for specific day to day planning, teachers jot down notes about **daily observations** on any of the following:
  - how the students use the problem-solving tools they selected
  - students' problem-solving strategies *(See sample forms in Appendix.)*
  - sizes of the numbers used in the problems
  - language students use to explain their solution strategies

- ☑ To show growth over time, teachers and students collect *samples* of students' work

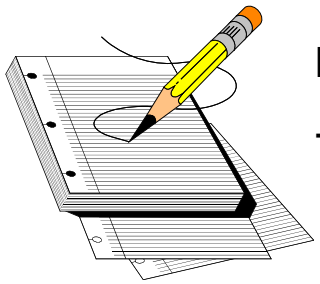
- ☑ To gather information on students' prior knowledge and skills about a specific topic, teachers use *inventories, student brainstorming lists, and informal pre-tests.*

- ☑ To have a record indicating growth of students' achievement based on differentiated instruction of a specific topic, teachers use *inventories, student brainstorming lists, and informal post-tests.*

- ☑ To illuminate a student's sense of number relationships and thinking strategies, teachers use **Fact Interviews** found in this chapter. *Completed Fact Interviews are also used to record student progress over time. Teachers pass completed Fact Interviews on to the following teacher each year.*

- ☑ To determine a student's understanding of each problem type and the strategies used for a sequence of number sizes, teachers use the **Problem Solving Interviews** found in this chapter. *Completed Problem Solving Interviews are also used to record student progress over time. Teachers pass completed Problem Solving Interviews on to the following teacher each year.*

- ☑ To have an overview of progress from year to year, teachers use the **P**rimary **M**ath **A**ssessments.



Fact Interviews

There are six interviews:

1. **Addition Fact Interview A**
   Sums Within and To Ten
   (First Grade MMSD Math Standard)

2. **Addition Fact Interview B**
   Sums to 20
   (Second Grade MMSD Math Standard)

3. **Subtraction Fact Interview C**
   Differences Less Than, Equal To, and Greater Than 3
   (Third Grade MMSD Math Standard)

4. **Multiplication Fact Interview C**
   2, 5, 4, and 3 As Multiplier or Multiplicand
   (Third Grade MMSD Math Standard)

5. **Multiplication Fact Interview D**
   All Multiplication Facts
   (Fourth Grade MMSD Math Standard)

6. **Division Fact Interview E**
   All Division Facts
   (Fifth Grade MMSD Math Standard)

Within each interview, the individual computations (facts) are organized developmentally from those requiring the least amount of mental number sense to those requiring the most flexibility in working with number relationships.

All six Fact Interviews are included in *Learning Mathematics in the Primary Grades*. Kindergarten, first and second grade teachers will most often use **Addition Fact Interview A and Addition Fact Interview B**. At times, they may have a student who is thinking abstractly about the other operations. Teachers use some or all of the other available Fact Interviews as needed.

Kindergarten students most often count everything, sometimes using fingers to show and count each set, in order to do single digit calculations. In first grade, most students develop counting on strategies. In second grade, students use number relationship strategies. **Addition Fact Interview A** and **Addition Fact Interview B** are used to record this development over time.

There will always be some kindergarten, first and second grade students who are developing number relationship strategies beyond addition. They understand that subtraction means finding the difference between two numbers as well as taking away one number from another. Teachers who want to monitor these students' development of number sense and strategies for computing basic single digit subtraction facts can use **Subtraction Fact Interview C**.

Some primary students may use their number sense to recognize sets of numbers within a larger group. These students are developing a multiplicative understanding of number. They are using number relationships to multiply instead of repeatedly adding a given number or skip counting. Teachers use **Multiplication Fact Interview C** and **Multiplication Fact Interview D** to identify a student's facility in working with groups.

A few primary students who have a well-developed understanding of multiplication might be working on division. **Division Fact Interview E** can be used to assess the strategies a student uses to compute basic division facts.

Purposes of Fact Interview Assessments:

- ☑ illuminate a child's sense of number relationships and the thinking strategies he/she uses to calculate single digit computations
- ☑ identify the size of numbers that a student can compute mentally (a student's mental computation level)
- ☑ identify the number sizes to use in number work, problem solving and inspecting equations activities
- ☑ identify which facts to use for a student's fluency and maintenance independent practice
- ☑ communicate a student's progress with parents and future teachers
- ☑ monitor a student's progress over time (Schools might want to keep the Fact Interviews in the blue PLAA binder.)

Materials:

- ☑ a student copy of the interview (without codes at the bottom)
- ☑ a teacher copy of the interview (with codes at the bottom)
- ☑ one pencil for the teacher, no pencil for the student
- ☑ no counters! (This interview is meant to determine a student's mental computation level.)

Conducting the interview:

The teacher begins with an interview that is at a student's independent level and continues until the student consistently uses a counting by ones strategy or a repeated addition or skip counting strategy. (This is similar to continuing a reading running record until reaching a level that challenges a student's word identification and comprehension strategies.)

- ☑ Find a place where you and the student can sit next to each other.
- ☑ Have a copy of the interview in front of the student and a copy in front of you.
- ☑ Say: ***"Please keep your hands above the table. I can learn about your math thinking if I can see when and how you use your fingers to help you."***
- ☑ Say: ***"Look at each equation. All you have to say out loud is the number that goes on each line."***
- ☑ Say: ***"Begin here,"*** (point to the equation at the top of the column on the left) ***and go down the column."***
- ☑ Do not read the equation aloud. If the student reads the equation aloud, you might want to remind him/her that the only thing he/she needs to say out loud is the number that goes on the line. Reading the entire equation out loud can be very tiring for the student!
- ☑ Write the number the student says on the line in each equation.
- ☑ Use the coding system (see the coding guides for each interview) to indicate the student's strategies.
- ☑ Continue the interview while the student consistently uses number relationship strategies.
- ☑ Stop the interview when you see a student consistently using counting strategies (all four operations) or a repeated addition strategy (multiplication and division).



Note:

For sums within 10, a child might count fingers as a matter of habit. Check a few sums across 10 to see if the child can use a counting on strategy.



Note:

Teachers learn students' number relationship strategies by asking students to explain their thinking on selected facts. See the coding guides for suggestions.

- ☑ If a student is using more than 3-4 seconds of thinking time, watch the student very carefully. If you can determine that he/she is using a counting strategy (mouthing numbers, nodding, tapping a foot, moving fingers slightly), code it as a counting strategy.
- ☑ If a student consistently responds within 3-4 seconds and you see no indications of a counting strategy, stop at specific facts and ask how he/she is thinking about the numbers to be able to compute so quickly. If he/she shares a counting strategy, note it with a code (see the coding guides).
- ☑ If he/she explains the use of a doubles or another known fact, a decomposing strategy, a compensating strategy, or various strategies for multiplication and division, write the expression that indicates the student's thinking above the equation.
- ☑ Finish the interview by saying, ***"Thank you for sharing your thinking with me."***
- ☑ Count all the facts that have no coding as well as those that indicate a strategy other than a counting strategy. Write the total and date on the line in the lower right corner.
- ☑ Complete the class roster if you want a class composite to aid in planning for instruction and evaluating growth.

Coding student responses: Counting Strategies



Use the following codes to record student's strategies. The codes indicate one of three things:

1. a student's counting strategy
2. a student uses more than 3-4 seconds of thinking time
3. the solution is inaccurate.

As you listen to the student's responses, use the following codes to mark each equation.

Coding Addition Fact Interviews A and B		
Code	**Example**	**Indicates**
ce	ce **4 + 5 = 9**	***c****ounts* ***e****verything* student puts up fingers for each addend and counts total
sb	sb **4 + 5 = 9**	***s****hows* ***b****oth sets* student puts up group of fingers for each addend and does not count total
f	f **4 + 5 = 9**	***f****ingers* student counts up from number with f written above it and puts up fingers to track counting
c	c **4 + 5 = 9**	***c****ounts* student counts on from the number with a c above it, perhaps tapping, nodding, or mouthing the count
•	**4 + 5 = 9.**	*dot after sum* student uses thinking time to respond, more than 3-4 seconds
—	**4 + 5 = 8**	*line above sum* response is incorrect

Coding student responses: Number Relationship Strategies

Teachers want to know which number relationship strategies a student uses. If you cannot see any evidence that a student is using a counting strategy, you should ask the student to explain his/her thinking. Select a near double (e.g., 6 + 5), an adding 9, (e.g., 7 + 9), or a making a 10 (e.g., 8 + 6). Ask the student: ***"Tell me how you are thinking about the numbers in order to do this one,"*** pointing to a specific equation**.** If the student shares a counting strategy, note it with the codes on the previous page. If the student shares a number relationship strategy, such as decomposing addends, compensating 9, or using a known fact, write the symbols that show the student's thinking above the equation.

Here are three examples:

Coding Addition Fact Interviews A and B	
6 + 6 + 2 **6 + 8 = 14**	student explains that he/she used a doubles fact (6+6+2)
8 + 2 + 4 **6 + 8 = 14**	student explains that he/she decomposed an addend (8+2+4)
6 + 10 - 1 **6 + 9 = 15**	student explains that he/she changed an addend and then subtracted to compensate for the change

Definitions:

- **Addend**—numbers that are added together
- **Sum**—total or whole amount, the result of adding
- **Compensating strategy**—changes the numbers in the equation, computes, and then adjusts the solution to correct for the change made at the beginning. E.g., given 6 + 9, the student shares "6 + 10 is 16. I have to take away 1 from the 16 because I added 1 to the 9."
- **Decomposing an addend**—uses another name for the quantity to make the computation more easily accomplished. E.g., given 6 + 8, the student shares "6 is 2 + 4. 8 + 2 is 10. 10 and 4 more is 14."

Coding student responses: Counting Strategies



Use the following codes to record student's strategies. The codes indicate one of three things:

1. a student's counting strategy
2. a student uses more than 3-4 seconds of thinking time
3. the solution is inaccurate.

As you listen to the student's responses, use the following codes to mark each equation.

Coding Subtraction Fact Interview C		
Code	**Example**	**Indicates**
ce	ce **9 – 6 = 3**	***c**ounts **e**verything* student puts up fingers for minuend and puts down a finger at a time for subtrahend student
sb	sb **9 – 6 = 3**	***s**hows **b**oth* student puts up group of fingers for minuend, puts down group of fingers for subtrahend, does not count remaining fingers
fb	fb **9 – 6 = 3**	***f**ingers **b**ack* student counts back the quantity indicated by the subtrahend, putting up fingers to track the counting (example: student put fingers up while saying 8, 7, 6, 5, 4, 3)
cb	cb **9 – 6 = 3**	***c**ounts **b**ack* student counts back the quantity indicated by the subtrahend (example: student said 8, 7, 6, 5, 4, 3)
f	f **9 – 6 = 3**	***f**ingers* student counts down from minuend to subtrahend or up from number with f written above it and puts up fingers to track counting (example: student put fingers up while saying 7, 8, 9)
c	c **9 – 6 = 3**	***c**ounts* student counts down or up from number with c written above it, perhaps tapping, nodding, or mouthing the count (example: student said 7, 8, 9)
•	**9 – 6 = 3 •**	*dot after difference* student uses thinking time to respond, more than 3-4 seconds
‾	‾ **9 – 6 = 4**	*line above difference* response is incorrect

Coding student responses: Number Relationship Strategies

Teachers want to know which number relationship strategies a student uses. If you cannot see any evidence that a student is using a counting strategy, you should ask the student to explain his/her thinking. Select an equation that might be solved by using a known fact (e.g., 13 - 6). Ask the student: ***"Tell me how you are thinking about the numbers in order to do this one,"*** pointing to the specific equation**.** You will also want to check on an equation that might be solved by working to 10 (e.g., 14 - 6). If the student shares a counting strategy, note it with the codes on the previous page. If the student shares a number relationship strategy, such as one of those listed below, write the symbols that show the student's thinking above the equation.

Here are four examples:

Coding Subtraction Fact Interview C	
6 + 8 = 14 **14 – 6 = 8**	student explains that he/she used a known addition fact
6 + ? = 14 **14 – 6 = 8**	student explains that he/she thought of the computation as determining a missing addend
14 – 4 – 2 **14 – 6 = 8**	student explains that he/she decomposed the subtrahend
10 – 6 + 4 **14 – 6 = 8**	student explains that he/she decomposed the minuend

Definitions:

- **Minuend**—the number being subtracted from
- **Subtrahend**—the number being subtracted
- **Difference**—the number that is left after one quantity is taken away from another, the result of subtracting

Coding student responses: Counting Strategies



Use the following codes to record student's strategies. The codes indicate one of three things:

1. a student's counting strategy
2. a student uses more than 3-4 seconds of thinking time
3. the solution is inaccurate.

As you listen to the student's responses, use the following codes to mark each equation.

Coding Multiplication Fact Interviews C and D		
Code	**Example**	**Indicates**
rc	rC (18) **8 x 6 = 48**	***r****epeatedly adds a few, then* ***c****ounts on* student repeatedly adds a few groups (i.e. 6, 12, 18), then counts on by ones (i.e. 19, 20, 21, 22, 23, 24 and so on)
r	r **8 x 6 = 48**	***r****epeatedly adds* student adds groups without counting
sf	sf **8 x 6 = 48**	***s****kip counts,* ***f****ingers* student skip counts, keeping track of groups with fingers
s	s **8 x 6 = 48**	***s****kip counts* student skip counts
•	**8 x 6 = 48 •**	*dot after product* student uses thinking time to respond, more than 3-4 seconds
—	**8 x 6 = 45**	*line above product* response is incorrect

Coding student responses: Number Sense Strategies

Teachers want to know which number relationship strategies a student uses. If you cannot see any evidence that a student is using a counting strategy, you should ask the student to explain her/his thinking.

- For **Interview C** – Multiplication, you might want to focus on **6 x 5, 8 x 4 and 9 x 3**.
- For **Interview D** – Multiplication, you might want to focus on **8 x 6, 9 x 7, and 7 x 5**.

Ask the student: **"Tell me how you are thinking about the numbers in order to do this one."** Point to 6 x 5 = ___ on Interview C or point to 8 x 6 = ___ on Interview D.
If the student shares a counting strategy, note it with the codes listed on the previous page.
If the student shares a number sense strategy, such as halving then doubling, using partial products, working from a known fact, or some other strategy, write the symbols that show the student's thinking above the equation.

Here are three examples:

Coding Multiplication Fact Interviews C and D	
24 + 24 **8 x 6 = 48**	student explains that he/she computed half and then doubled (i.e. student computed 4 x 6 and then doubled that product)
30 + 18 **8 x 6 = 48**	student explains that he/she added partial products (i.e. student computed 5 x 6 and 3 x 6 and added their products)
60 - 6 **9 x 6 = 54**	student explains that he/she worked from a known fact

Definitions:

- **Multiplicand**—the number (factor) being multiplied
- **Multiplier**—the number (factor) being multiplied by
- **Product**—the result of multiplying
- **Factor**—one of the whole numbers multiplied to get a given number; an integer that divides evenly into an integer

Coding student responses: Counting Strategies



Use the following codes to record student's strategies. The codes indicate one of three things:

1. a student's counting strategy
2. a student uses more than 3-4 seconds of thinking time
3. the solution is inaccurate.

As you listen to the student's responses, use the following codes to mark each equation.

Coding Division Fact Interview E		
Code	**Example**	**Indicates**
rs	r **24 ÷ 6 = 4**	**r**epeatedly **s**ubtracts student repeatedly subtracts the divisor
ra	ra **24 ÷ 6 = 4**	**r**epeatedly **a**dds student repeatedly adds the divisor
s	s **24 ÷ 6 = 4**	**s**kip counts student skip counts
•	**24 ÷ 6 = 4•**	dot after quotient student uses thinking time to respond, more than 3-4 seconds
—	— **24 ÷ 6 = 3**	line above quotient response is incorrect

Coding student responses: Number Relationship Strategies

Teachers want to know which number relationship strategies a student uses. If you cannot see any evidence that a student is using a counting strategy, you should ask the student to explain his/her thinking. Select an equation that might be solved by using a known fact (e.g., 24 ÷ 6). Ask the student: ***"Tell me how you are thinking about the numbers in order to do this one,"*** pointing to the specific equation. If the student shares a counting strategy, note it with the codes on the previous page. If the student shares a number relationship strategy, such as one of those listed below, write the symbols that show the student's thinking above the equation.
Here are two examples:

? × 6 = 24 **24 ÷ 6 = 4**	student explains that he/she thought "What times 6 is 24?"
6 × 4 **24 ÷ 6 = 4**	student explains that he/she used known multiplication fact

Definitions:

- **Dividend**—the quantity to be divided
- **Divisor**—the quantity by which another quantity is to be divided
- **Quotient**—the result of dividing one quantity by another

Interview A - Addition

1 + 1 = ____	5 + 5 = ____	3 + 2 = ____	5 + 3 = ____	3 + 5 = ____
5 + 1 = ____	1 + 2 = ____	4 + 0 = ____	2 + 8 = ____	7 + 3 = ____
3 + 1 = ____	1 + 4 = ____	5 + 2 = ____	3 + 4 = ____	2 + 6 = ____
8 + 1 = ____	1 + 3 = ____	2 + 3 = ____	0 + 9 = ____	2 + 4 = ____
2 + 2 = ____	1 + 7 = ____	7 + 2 = ____	6 + 3 = ____	6 + 4 = ____
7 + 1 = ____	1 + 5 = ____	0 + 5 = ____	4 + 3 = ____	2 + 7 = ____
6 + 1 = ____	3 + 3 = ____	4 + 2 = ____	5 + 4 = ____	3 + 7 = ____
2 + 1 = ____	1 + 8 = ____	6 + 2 = ____	3 + 6 = ____	1 + 9 = ____
4 + 1 = ____	1 + 6 = ____	8 + 2 = ____	6 + 0 = ____	4 + 6 = ____
9 + 1 = ____	4 + 4 = ____	2 + 5 = ____	4 + 5 = ____	10 + 0 = ____

Addition Fact Interview A - Sums Within and To 10 Name_____

1 + 1 = ____	5 + 5 = ____	3 + 2 = ____	5 + 3 = ____	3 + 5 = ____
5 + 1 = ____	1 + 2 = ____	4 + 0 = ____	2 + 8 = ____	7 + 3 = ____
3 + 1 = ____	1 + 4 = ____	5 + 2 = ____	3 + 4 = ____	2 + 6 = ____
8 + 1 = ____	1 + 3 = ____	2 + 3 = ____	0 + 9 = ____	2 + 4 = ____
2 + 2 = ____	1 + 7 = ____	7 + 2 = ____	6 + 3 = ____	6 + 4 = ____
7 + 1 = ____	1 + 5 = ____	0 + 5 = ____	4 + 3 = ____	2 + 7 = ____
6 + 1 = ____	3 + 3 = ____	4 + 2 = ____	5 + 4 = ____	3 + 7 = ____
2 + 1 = ____	1 + 8 = ____	6 + 2 = ____	3 + 6 = ____	1 + 9 = ____
4 + 1 = ____	1 + 6 = ____	8 + 2 = ____	6 + 0 = ____	4 + 6 = ____
9 + 1 = ____	4 + 4 = ____	2 + 5 = ____	4 + 5 = ____	10 + 0 = ____

Coding: **ce** – counted everything; **sb** - showed both sets, doesn't count all; **f** – used fingers to count on from; **c** - counted on from; **dot** after sum – used thinking time; **line** above sum – incorrect

Notes:

Recorder _____Date _____ Score ____/50
Recorder _____Date _____ Score ____/50

Addition Fact Interview A Data First Grade Teacher _____ Date _____

Name	Within and To 10 Facts						Across 10 Facts								
	Counts both sets	Fingers		Counts on		Recalls	Fingers		Counts on		Uses Number Relationship Strategies				Recalls
		From First	From largest	From first	From largest		From first	From largest	From first	From largest	Uses a double	Decomposes to make 10	Compensates	Uses known fact	

Knows Number Sense Strategies for Addition Facts (Level A)

	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
0	0 + 0	1 + 0	2 + 0	3 + 0	4 + 0	5 + 0	6 + 0	7 + 0	8 + 0	9 + 0	10 + 0
1	0 + 1	1 + 1	2 + 1	3 + 1	4 + 1	5 + 1	6 + 1	7 + 1	8 + 1	9 + 1	10 + 1
2	0 + 2	1 + 2	2 + 2	3 + 2	4 + 2	5 + 2	6 + 2	7 + 2	8 + 2		10 + 2
3	0 + 3	1 + 3	2 + 3	3 + 3	4 + 3	5 + 3	6 + 3	7 + 3			10 + 3
4	0 + 4	1 + 4	2 + 4	3 + 4	4 + 4	5 + 4	6 + 4				10 + 4
5	0 + 5	1 + 5	2 + 5	3 + 5	4 + 5	5 + 5					10 + 5
6	0 + 6	1 + 6	2 + 6	3 + 6	4 + 6						10 + 6
7	0 + 7	1 + 7	2 + 7	3 + 7							10 + 7
8	0 + 8	1 + 8	2 + 8								10 + 8
9	0 + 9	1 + 9									10 + 9
10	0 + 10	1 + 10	2 + 10	3 + 10	4 + 10	5 + 10	6 + 10	7 + 10	8 + 10	9 + 10	

Name _____ Date Started _____ Goal Met _____

Interview B - Addition

5 + 1 = ____	3 + 3 = ____	4 + 6 = ____	6 + 8 = ____	5 + 7 = ____
1 + 7 = ____	3 + 4 = ____	5 + 5 = ____	3 + 9 = ____	5 + 9 = ____
1 + 9 = ____	3 + 6 = ____	3 + 7 = ____	9 + 3 = ____	8 + 6 = ____
4 + 2 = ____	0 + 9 = ____	2 + 8 = ____	8 + 7 = ____	9 + 9 = ____
2 + 5 = ____	3 + 5 = ____	6 + 6 = ____	7 + 7 = ____	4 + 9 = ____
3 + 2 = ____	4 + 4 = ____	2 + 9 = ____	6 + 7 = ____	8 + 4 = ____
2 + 4 = ____	5 + 3 = ____	7 + 4 = ____	7 + 9 = ____	6 + 9 = ____
2 + 6 = ____	4 + 3 = ____	5 + 6 = ____	9 + 8 = ____	7 + 8 = ____
2 + 3 = ____	4 + 5 = ____	3 + 8 = ____	8 + 8 = ____	9 + 7 = ____
2 + 7 = ____	6 + 4 = ____	7 + 5 = ____	8 + 5 = ____	5 + 8 = ____

Addition Fact Interview B - Sums To 20—Within, To, and Across 10 Name_____

5 + 1 = ____	3 + 3 = ____	4 + 6 = ____	6 + 8 = ____	5 + 7 = ____
1 + 7 = ____	3 + 4 = ____	5 + 5 = ____	3 + 9 = ____	5 + 9 = ____
1 + 9 = ____	3 + 6 = ____	3 + 7 = ____	9 + 3 = ____	8 + 6 = ____
4 + 2 = ____	0 + 9 = ____	2 + 8 = ____	8 + 7 = ____	9 + 9 = ____
2 + 5 = ____	3 + 5 = ____	6 + 6 = ____	7 + 7 = ____	4 + 9 = ____
3 + 2 = ____	4 + 4 = ____	2 + 9 = ____	6 + 7 = ____	8 + 4 = ____
2 + 4 = ____	5 + 3 = ____	7 + 4 = ____	7 + 9 = ____	6 + 9 = ____
2 + 6 = ____	4 + 3 = ____	5 + 6 = ____	9 + 8 = ____	7 + 8 = ____
2 + 3 = ____	4 + 5 = ____	3 + 8 = ____	8 + 8 = ____	9 + 7 = ____
2 + 7 = ____	6 + 4 = ____	7 + 5 = ____	8 + 5 = ____	5 + 8 = ____

Coding: **ce** – counted everything; **sb** - showed both sets, doesn't count all; **f** – used fingers to count on from; **c** - counted on from; **dot** after sum – used thinking time; **line** above sum – incorrect

Notes:

Recorder _____Date _____ Score ____/50
Recorder _____Date _____ Score ____/50

Addition Fact Interview B Data Second Grade Teacher _____ Date _____

Name	Within and To 10 Facts						Across 10 Facts								
	Counts both sets	Fingers		Counts on		Recalls	Fingers		Counts on		Uses Number Relationship Strategies				Recalls
		From First	From largest	From first	From largest		From first	From largest	From first	From largest	Uses a double	Decomposes to make 10	Compensates	Uses known fact	

Knows Number Sense Strategies for Addition Facts (Level B)

	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
0	0 + 0	1 + 0	2 + 0	3 + 0	4 + 0	5 + 0	6 + 0	7 + 0	8 + 0	9 + 0	10 + 0
1	0 + 1	1 + 1	2 + 1	3 + 1	4 + 1	5 + 1	6 + 1	7 + 1	8 + 1	9 + 1	10 + 1
2	0 + 2	1 + 2	2 + 2	3 + 2	4 + 2	5 + 2	6 + 2	7 + 2	8 + 2	9 + 2	10 + 2
3	0 + 3	1 + 3	2 + 3	3 + 3	4 + 3	5 + 3	6 + 3	7 + 3	8 + 3	9 + 3	10 + 3
4	0 + 4	1 + 4	2 + 4	3 + 4	4 + 4	5 + 4	6 + 4	7 + 4	8 + 4	9 + 4	10 + 4
5	0 + 5	1 + 5	2 + 5	3 + 5	4 + 5	5 + 5	6 + 5	7 + 5	8 + 5	9 + 5	10 + 5
6	0 + 6	1 + 6	2 + 6	3 + 6	4 + 6	5 + 6	6 + 6	7 + 6	8 + 6	9 + 6	10 + 6
7	0 + 7	1 + 7	2 + 7	3 + 7	4 + 7	5 + 7	6 + 7	7 + 7	8 + 7	9 + 7	10 + 7
8	0 + 8	1 + 8	2 + 8	3 + 8	4 + 8	5 + 8	6 + 8	7 + 8	8 + 8	9 + 8	10 + 8
9	0 + 9	1 + 9	2 + 9	3 + 9	4 + 9	5 + 9	6 + 9	7 + 9	8 + 9	9 + 9	10 + 9
10	0 + 10	1 + 10	2 + 10	3 + 10	4 + 10	5 + 10	6 + 10	7 + 10	8 + 10	9 + 10	10 + 10

Name _____ Date Started _____ Goal Met _____

Interview C - Subtraction

2 – 1 = ____	8 – 6 = ____	8 – 3 = ____	14 – 7 = ____	16 – 8 = ____
4 – 3 = ____	9 – 7 = ____	10 – 5 = ____	11 – 7 = ____	12 – 5 = ____
6 – 5 = ____	10 – 8 = ____	10 – 4 = ____	12 – 3 = ____	18 – 9 = ____
8 – 8 = ____	6 – 3 = ____	10 – 6 = ____	12 – 8 = ____	16 – 9 = ____
9 – 8 = ____	9 – 6 = ____	10 – 3 = ____	13 – 5 = ____	13 – 9 = ____
8 – 7 = ____	7 – 4 = ____	12 – 6 = ____	17 – 9 = ____	17 – 8 = ____
4 – 2 = ____	8 – 5 = ____	11 – 9 = ____	13 – 6 = ____	14 – 9 = ____
5 – 3 = ____	10 – 7 = ____	11 – 6 = ____	12 – 4 = ____	12 – 7 = ____
7 – 5 = ____	8 – 4 = ____	11 – 5 = ____	14 – 8 = ____	15 – 8 = ____
6 – 4 = ____	9 – 4 = ____	11 – 8 = ____	15 – 7 = ____	16 – 7 = ____

Subtraction Fact Interview C - Differences Less Than, Equal To, and Greater Than 3 Name_____

2 – 1 = ____	8 – 6 = ____	8 – 3 = ____	14 – 7 = ____	16 – 8 = ____
4 – 3 = ____	9 – 7 = ____	10 – 5 = ____	11 – 7 = ____	12 – 5 = ____
6 – 5 = ____	10 – 8 = ____	10 – 4 = ____	12 – 3 = ____	18 – 9 = ____
8 – 8 = ____	6 – 3 = ____	10 – 6 = ____	12 – 8 = ____	16 – 9 = ____
9 – 8 = ____	9 – 6 = ____	10 – 3 = ____	13 – 5 = ____	13 – 9 = ____
8 – 7 = ____	7 – 4 = ____	12 – 6 = ____	17 – 9 = ____	17 – 8 = ____
4 – 2 = ____	8 – 5 = ____	11 – 9 = ____	13 – 6 = ____	14 – 9 = ____
5 – 3 = ____	10 – 7 = ____	11 – 6 = ____	12 – 4 = ____	12 – 7 = ____
7 – 5 = ____	8 – 4 = ____	11 – 5 = ____	14 – 8 = ____	15 – 8 = ____
6 – 4 = ____	9 – 4 = ____	11 – 8 = ____	15 – 7 = ____	16 – 7 = ____

Coding: **ce** – counted whole, took away, then counted remaining set; **sb** – showed sets, didn't count; **fb** – used fingers to count back subtrahend; **cb** counted back subtrahend; **f** – used fingers to count difference to or from; **c** – counted difference up from or down to; **dot** after difference – used thinking time; **line** above difference – incorrect

Notes:

Recorder _____Date _____ Score ____/50

Recorder _____Date _____ Score ____/50

Subtraction Fact Interview C Data Third Grade Teacher _____ Date _____

Name	Facts With a Difference of 1, 2, or 3								Facts With a Difference of 4 or more											
		Fingers			Counts				Fingers			Counts			Uses Number Relationship Strategies					
	Counts all	Back from	Back to	Up from	Back from	Back to	Up from	Recalls	Back from	Back to	Up from	Back from	Back to	Up from	Uses known addition fact	Thinks of missing addend	Decom-poses minuend	Decom-poses subtra-hend	Compen-sates	Recalls

Knows Number Sense Strategies for Subtraction Facts (Level C)

Difference of 0	Difference of 1	Difference of 2	Difference of 3	Difference of 4	Difference of 5	Difference of 6	Difference of 7	Difference of 8	Difference of 9	Difference of 10
0 – 0	1 – 0	2 – 0	3 – 0	4 – 0	5 – 0	6 – 0	7 – 0	8 – 0	9 – 0	10 – 0
1 – 1	2 – 1	3 – 1	4 – 1	5 – 1	6 – 1	7 – 1	8 – 1	9 – 1	10 – 1	11 – 1
2 – 2	3 – 2	4 – 2	5 – 2	6 – 2	7 – 2	8 – 2	9 – 2	10 – 2	11 – 2	12 – 2
3 – 3	4 – 3	5 – 3	6 – 3	7 – 3	8 – 3	9 – 3	10 – 3	11 – 3	12 – 3	13 – 3
4 – 4	5 – 4	6 – 4	7 – 4	8 – 4	9 – 4	10 – 4	11 – 4	12 – 4	13 – 4	14 – 4
5 – 5	6 – 5	7 – 5	8 – 5	9 – 5	10 – 5	11 – 5	12 – 5	13 – 5	14 – 5	15 – 5
6 – 6	7 – 6	8 – 6	9 – 6	10 – 6	11 – 6	12 – 6	12 – 6	14 – 6	15 – 6	16 – 6
7 – 7	8 – 7	9 – 7	10 – 7	11 – 7	12 – 7	13 – 7	14 – 7	15 – 7	16 – 7	17 – 7
8 – 8	9 – 8	10 – 8	11 – 8	12 – 8	13 – 8	14 – 8	15 – 8	16 – 8	17 – 8	18 – 8
9 – 9	10 – 9	11 – 9	12 – 9	13 – 9	14 – 9	15 – 9	16 – 9	17 – 9	18 – 9	19 – 9
10 – 10	11 – 10	12 – 10	13 – 10	14 – 10	15 – 10	16 – 10	17 – 10	18 – 10	19 – 10	20 – 10

Name _____ Date Started _____ Goal Met _____

Interview C - Multiplication

1 x 5 = ____	6 x 2 = ____	5 x 5 = ____	0 x 6 = ____	4 x 9 = ____
2 x 3 = ____	2 x 9 = ____	5 x 3 = ____	4 x 4 = ____	6 x 3 = ____
3 x 0 = ____	2 x 7 = ____	6 x 5 = ____	7 x 4 = ____	7 x 3 = ____
2 x 4 = ____	7 x 2 = ____	5 x 6 = ____	6 x 4 = ____	4 x 8 = ____
7 x 1 = ____	8 x 2 = ____	5 x 4 = ____	4 x 3 = ____	8 x 3 = ____
5 x 2 = ____	2 x 8 = ____	5 x 8 = ____	4 x 6 = ____	3 x 6 = ____
2 x 6 = ____	9 x 2 = ____	9 x 5 = ____	4 x 7 = ____	3 x 8 = ____
2 x 5 = ____	3 x 5 = ____	5 x 9 = ____	8 x 4 = ____	9 x 3 = ____
3 x 2 = ____	4 x 5 = ____	5 x 7 = ____	3 x 3 = ____	3 x 7 = ____
4 x 2 = ____	8 x 5 = ____	7 x 5 = ____	9 x 4 = ____	3 x 9 = ____

Multiplication Fact Interview C - 2, 5, 4, and 3 As Multiplier or Multiplicand Name_____

1 x 5 = ____	6 x 2 = ____	5 x 5 = ____	0 x 6 = ____	4 x 9 = ____
2 x 3 = ____	2 x 9 = ____	5 x 3 = ____	4 x 4 = ____	6 x 3 = ____
3 x 0 = ____	2 x 7 = ____	6 x 5 = ____	7 x 4 = ____	7 x 3 = ____
2 x 4 = ____	7 x 2 = ____	5 x 6 = ____	6 x 4 = ____	4 x 8 = ____
7 x 1 = ____	8 x 2 = ____	5 x 4 = ____	4 x 3 = ____	8 x 3 = ____
5 x 2 = ____	2 x 8 = ____	5 x 8 = ____	4 x 6 = ____	3 x 6 = ____
2 x 6 = ____	9 x 2 = ____	9 x 5 = ____	4 x 7 = ____	3 x 8 = ____
2 x 5 = ____	3 x 5 = ____	5 x 9 = ____	8 x 4 = ____	9 x 3 = ____
3 x 2 = ____	4 x 5 = ____	5 x 7 = ____	3 x 3 = ____	3 x 7 = ____
4 x 2 = ____	8 x 5 = ____	7 x 5 = ____	9 x 4 = ____	3 x 9 = ____

Coding: **rc**-repeatedly added, then counted on; **r**-repeatedly added; **sf**-used fingers to track skip counting, **s**-skip counted; **dot** after product-used thinking time; **line** above product-incorrect

Notes:

Recorder _____Date _____ Score ____/50
Recorder _____Date _____ Score ____/50

Multiplication Fact Interview C Data Third Grade Teacher _____ Date _____

Name	2, 5, 4 and 3 as Multiplier or Multiplicand								
	Counts everything	Adds		Skip Counts		**Uses Number Relationship Strategies**			Recalls
		Adds a few, then counts on	Repeatedly adds	Uses fingers to track skip counting	Uses memorized sequence	Uses doubling	Works from a known fact	Adds partial products	

Knows Number Sense Strategies for Multiplication Facts (Level C)

	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
0	0 x 0	1 x 0	2 x 0	3 x 0	4 x 0	5 x 0	6 x 0	7 x 0	8 x 0	9 x 0	10 x 0
1	0 x 1	1 x 1	2 x 1	3 x 1	4 x 1	5 x 1	6 x 1	7 x 1	8 x 1	9 x 1	10 x 1
2	0 x 2	1 x 2	2 x 2	3 x 2	4 x 2	5 x 2	6 x 2	7 x 2	8 x 2	9 x 2	10 x 2
3	0 x 3	1 x 3	2 x 3	3 x 3	4 x 3	5 x 3	6 x 3	7 x 3	8 x 3	9 x 3	10 x 3
4	0 x 4	1 x 4	2 x 4	3 x 4	4 x 4	5 x 4	6 x 4	7 x 4	8 x 4	9 x 4	10 x 4
5	0 x 5	1 x 5	2 x 5	3 x 5	4 x 5	5 x 5	6 x 5	7 x 5	8 x 5	9 x 5	10 x 5
6	0 x 6	1 x 6	2 x 6	3 x 6	4 x 6	5 x 6					
7	0 x 7	1 x 7	2 x 7	3 x 7	4 x 7	5 x 7					
8	0 x 8	1 x 8	2 x 8	3 x 8	4 x 8	5 x 8					
9	0 x 9	1 x 9	2 x 9	3 x 9	4 x 9	5 x 9					
10	0 x 10	1 x 10	2 x 10	3 x 10	4 x 10	5 x 10					

Name _____ Date Started _____ Goal Met _____

Interview D - Multiplication

1 x 5 = ____	3 x 5 = ____	6 x 4 = ____	8 x 3 = ____	9 x 4 = ____
2 x 4 = ____	5 x 3 = ____	3 x 4 = ____	3 x 8 = ____	8 x 8 = ____
7 x 1 = ____	4 x 5 = ____	4 x 3 = ____	3 x 9 = ____	6 x 9 = ____
3 x 0 = ____	8 x 5 = ____	4 x 6 = ____	4 x 7 = ____	7 x 7 = _____
6 x 2 = ____	5 x 4 = ____	7 x 4 = ____	5 x 9 = ____	7 x 8 = ____
2 x 8 = ____	5 x 5 = ____	4 x 8 = ____	6 x 6 = ____	9 x 9 = ____
7 x 2 = ____	5 x 7 = ____	3 x 3 = ____	4 x 9 = ____	8 x 9 = ____
2 x 9 = ____	6 x 5 = ____	6 x 3 = ____	7 x 5 = ____	6 x 7 = ____
8 x 2 = ____	5 x 8 = ____	3 x 7 = ____	8 x 4 = ____	9 x 7 = ____
0 x 6 = ____	4 x 4 = ____	7 x 3 = ____	8 x 7 = ____	8 x 6 = ____

Multiplication Fact Interview D - All Multiplication Facts Name_____

1 x 5 = ____	3 x 5 = ____	6 x 4 = ____	8 x 3 = ____	9 x 4 = ____
2 x 4 = ____	5 x 3 = ____	3 x 4 = ____	3 x 8 = ____	8 x 8 = ____
7 x 1 = ____	4 x 5 = ____	4 x 3 = ____	3 x 9 = ____	6 x 9 = ____
3 x 0 = ____	8 x 5 = ____	4 x 6 = ____	4 x 7 = ____	7 x 7 = _____
6 x 2 = ____	5 x 4 = ____	7 x 4 = ____	5 x 9 = ____	7 x 8 = ____
2 x 8 = ____	5 x 5 = ____	4 x 8 = ____	6 x 6 = ____	9 x 9 = ____
7 x 2 = ____	5 x 7 = ____	3 x 3 = ____	4 x 9 = ____	8 x 9 = ____
2 x 9 = ____	6 x 5 = ____	6 x 3 = ____	7 x 5 = ____	6 x 7 = ____
8 x 2 = ____	5 x 8 = ____	3 x 7 = ____	8 x 4 = ____	9 x 7 = ____
0 x 6 = ____	4 x 4 = ____	7 x 3 = ____	8 x 7 = ____	8 x 6 = ____

Coding: **rc**-repeatedly added, then counted on; **r**-repeatedly added; **sf**-used fingers to track skip counting; **s**-skip counted; **dot** after product-used thinking time; **line** above product-incorrect

Notes:

Recorder _____Date _____ Score ___/50
Recorder _____Date _____ Score ___/50

Multiplication Fact Interview D Data Fourth Grade Teacher _____ Date _____

Name	All Multiplication Facts								
	Counts Everything	Adds		Skip Counts		**Uses Number relationship Strategies**			Recalls
		Adds a few, then counts on	Repeatedly adds	Uses fingers to track skip counting	Uses memorized sequence	Uses doubling	Works from a known fact	Adds partial products	

Knows Number Sense Strategies for Multiplication Facts (Level D)

	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
0	0 x 0	1 x 0	2 x 0	3 x 0	4 x 0	5 x 0	6 x 0	7 x 0	8 x 0	9 x 0	10 x 0
1	0 x 1	1 x 1	2 x 1	3 x 1	4 x 1	5 x 1	6 x 1	7 x 1	8 x 1	9 x 1	10 x 1
2	0 x 2	1 x 2	2 x 2	3 x 2	4 x 2	5 x 2	6 x 2	7 x 2	8 x 2	9 x 2	10 x 2
3	0 x 3	1 x 3	2 x 3	3 x 3	4 x 3	5 x 3	6 x 3	7 x 3	8 x 3	9 x 3	10 x 3
4	0 x 4	1 x 4	2 x 4	3 x 4	4 x 4	5 x 4	6 x 4	7 x 4	8 x 4	9 x 4	10 x 4
5	0 x 5	1 x 5	2 x 5	3 x 5	4 x 5	5 x 5	6 x 5	7 x 5	8 x 5	9 x 5	10 x 5
6	0 x 6	1 x 6	2 x 6	3 x 6	4 x 6	5 x 6	6 x 6	7 x 6	8 x 6	9 x 6	10 x 6
7	0 x 7	1 x 7	2 x 7	3 x 7	4 x 7	5 x 7	6 x 7	7 x 7	8 x 7	9 x 7	10 x 7
8	0 x 8	1 x 8	2 x 8	3 x 8	4 x 8	5 x 8	6 x 8	7 x 8	8 x 8	9 x 8	10 x 8
9	0 x 9	1 x 9	2 x 9	3 x 9	4 x 9	5 x 9	6 x 9	7 x 9	8 x 9	9 x 9	10 x 9
10	0 x 10	1 x 10	2 x 10	3 x 10	4 x 10	5 x 10	6 x 10	7 x 10	8 x 10	9 x 10	10 x 10

Name _____ Date Started _____ Goal Met _____

Interview E - Division

4 ÷ 2 = ____	40 ÷ 5 = ____	21 ÷ 3 = ____	56 ÷ 7 = ____	36 ÷ 9 = ____
10 ÷ 2 = ____	15 ÷ 5 = ____	32 ÷ 4 = ____	35 ÷ 7 = ____	48 ÷ 8 = ____
20 ÷ 4 = ____	30 ÷ 5 = ____	27 ÷ 3 = ____	27 ÷ 9 = ____	42 ÷ 7 = ____
14 ÷ 2 = ____	45 ÷ 5 = ____	24 ÷ 3 = ____	54 ÷ 6 = ____	64 ÷ 8 = ____
25 ÷ 5 = ____	12 ÷ 3 = ____	36 ÷ 4 = ____	36 ÷ 6 = ____	72 ÷ 8 = ____
18 ÷ 2 = ____	16 ÷ 4 = ____	30 ÷ 6 = ____	40 ÷ 8 = ____	63 ÷ 9 = ____
5 ÷ 5 = ____	24 ÷ 4 = ____	18 ÷ 6 = ____	56 ÷ 8 = ____	63 ÷ 7 = ____
16 ÷ 2 = ____	18 ÷ 3 = ____	28 ÷ 7 = ____	49 ÷ 7 = ____	81 ÷ 9 = ____
12 ÷ 6 = ____	9 ÷ 3 = ____	32 ÷ 8 = ____	42 ÷ 6 = ____	48 ÷ 6 = ____
35 ÷ 5 = ____	28 ÷ 4 = ____	21 ÷ 7 = ____	45 ÷ 9 = ____	54 ÷ 9 = ____

Division Fact Interview E - All Division Facts

Name_____

4 ÷ 2 = ____	40 ÷ 5 = ____	21 ÷ 3 = ____	56 ÷ 7 = ____	36 ÷ 9 = ____
10 ÷ 2 = ____	15 ÷ 5 = ____	32 ÷ 4 = ____	35 ÷ 7 = ____	48 ÷ 8 = ____
20 ÷ 4 = ____	30 ÷ 5 = ____	27 ÷ 3 = ____	27 ÷ 9 = ____	42 ÷ 7 = ____
14 ÷ 2 = ____	45 ÷ 5 = ____	24 ÷ 3 = ____	54 ÷ 6 = ____	64 ÷ 8 = ____
25 ÷ 5 = ____	12 ÷ 3 = ____	36 ÷ 4 = ____	36 ÷ 6 = ____	72 ÷ 8 = ____
18 ÷ 2 = ____	16 ÷ 4 = ____	30 ÷ 6 = ____	40 ÷ 8 = ____	63 ÷ 9 = ____
5 ÷ 5 = ____	24 ÷ 4 = ____	18 ÷ 6 = ____	56 ÷ 8 = ____	63 ÷ 7 = ____
16 ÷ 2 = ____	18 ÷ 3 = ____	28 ÷ 7 = ____	49 ÷ 7 = ____	81 ÷ 9 = ____
12 ÷ 6 = ____	9 ÷ 3 = ____	32 ÷ 8 = ____	42 ÷ 6 = ____	48 ÷ 6 = ____
35 ÷ 5 = ____	28 ÷ 4 = ____	21 ÷ 7 = ____	45 ÷ 9 = ____	54 ÷ 9 = ____

Coding: **rs** – repeatedly subtracted number, **ra** - repeatedly added number, **sf**-used fingers to track skip counting; **s** – skip counted, **dot** after quotient – used thinking time, **line** above quotient – incorrect

Notes:

Recorder _____Date _____ Score ____/50
Recorder _____Date _____ Score ____/50

Division Fact Interview E Data Fifth Grade Teacher _____ Date _____

Name	All Division Facts							
	Adds		Skip Counts		**Uses Number Relationship Strategies**			Recalls
	Adds a few, then counts on	Repeatedly adds	Uses fingers to track skip counting	Uses memorized sequence	Uses inverse multiplication fact	Works from a known division fact	Halves and then doubles	

Knows Number Sense Strategies for Division Facts (Level E)

Dividend of 0	Dividend of 1	Dividend of 2	Dividend of 3	Dividend of 4	Dividend of 5	Dividend of 6	Dividend of 7	Dividend of 8	Dividend of 9	Dividend of 10
1 ÷ 0	1 ÷ 1	2 ÷ 1	3 ÷ 1	4 ÷ 1	5 ÷ 1	6 ÷ 1	7 ÷ 1	8 ÷ 1	9 ÷ 1	10 ÷ 1
2 ÷ 0	2 ÷ 2	4 ÷ 2	6 ÷ 2	8 ÷ 2	10 ÷ 2	12 ÷ 2	14 ÷ 2	16 ÷ 2	18 ÷ 2	20 ÷ 2
3 ÷ 0	3 ÷ 3	6 ÷ 3	9 ÷ 3	12 ÷ 3	15 ÷ 3	18 ÷ 3	21 ÷ 3	24 ÷ 3	27 ÷ 3	30 ÷ 3
4 ÷ 0	4 ÷ 4	8 ÷ 4	12 ÷ 4	16 ÷ 4	20 ÷ 4	24 ÷ 4	28 ÷ 4	32 ÷ 4	36 ÷ 4	40 ÷ 4
5 ÷ 0	5 ÷ 5	10 ÷ 5	15 ÷ 5	20 ÷ 5	25 ÷ 5	30 ÷ 5	35 ÷ 5	40 ÷ 5	45 ÷ 5	50 ÷ 5
6 ÷ 0	6 ÷ 6	12 ÷ 6	18 ÷ 6	24 ÷ 6	30 ÷ 6	36 ÷ 6	42 ÷ 6	48 ÷ 6	54 ÷ 6	60 ÷ 6
7 ÷ 0	7 ÷ 7	14 ÷ 7	21 ÷ 7	28 ÷ 7	35 ÷ 7	42 ÷ 7	49 ÷ 7	56 ÷ 7	63 ÷ 7	70 ÷ 7
8 ÷ 0	8 ÷ 8	16 ÷ 8	24 ÷ 8	32 ÷ 8	40 ÷ 8	48 ÷ 8	56 ÷ 8	64 ÷ 8	72 ÷ 8	80 ÷ 8
9 ÷ 0	9 ÷ 9	18 ÷ 9	27 ÷ 9	36 ÷ 9	45 ÷ 9	54 ÷ 9	63 ÷ 9	72 ÷ 9	81 ÷ 9	90 ÷ 9
10 ÷ 0	10 ÷ 10	20 ÷ 10	30 ÷ 10	40 ÷ 10	50 ÷ 10	60 ÷ 10	70 ÷ 10	80 ÷ 10	90 ÷ 10	100 ÷ 10

Name _____ Date Started _____ Goal Met _____

Fact Computation Development **Student: _____ Date: _____**

	Uses counting strategies	Uses number relationships
Addition Facts		
Sums to 10		
Sums to 20		
Subtraction Facts		
Differences of 1, 2, 3		
Differences greater than 4		

	Uses counting or repeated addition strategies	Uses number relationships
Multiplication Facts		
2, 5, 4, 3 as multiplier or multiplicand		
6, 7, 8, 9 as multiplier or multiplicand		
Division Facts		
Divisors of 2, 5, 4, 3		
Divisors of 6, 7, 8, 9		

Use Number Relationships to Help Compute Basic Facts *

Big Ideas to Remember:

1. Number relationships can be used to help remember basic facts.
2. There are patterns and relationships in basic facts. You can figure out new or unknown facts from the ones you already know.
3. All the facts can be learned with the help of efficient strategies.

Steps to Take to Learn the Facts:

1. Develop a strong understanding of the operations and of number relationships.
2. Think of efficient strategies to help recall the facts.
3. Decide which strategies work best for particular numbers.
4. Practice using these strategies.

* Van de Walle, J. A. (2001). *Elementary and middle school mathematics*. New York, NY: Longman.

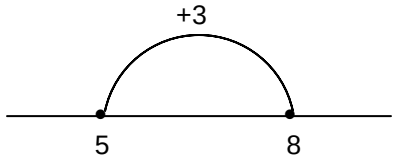
Try out these strategies!

Addition

- Use a double. If you know 7 + 7, use it to help you know 8 + 7 and 6 + 7.
- Look for doubles that might be hiding. For instance, 5 + 7 can be thought of as 5 + 5 + 2.
- Make it easier to add 9. Change the 9 to 10 by adding 1 to it (in your head). Then add the other set. To finish, take away 1 from the sum (to undo that change you made when you added 1 to 9 to change it to 10).
- Here's another way to add 9 quickly. Look at 9 + 5. Think of a name for 5 that uses a 1 (1 + 4)! Then add 9 + 1 to make 10 and finish by adding 4 to the 10.
- Make it easier to add 8. Look at 8 + 6. Think of a name for 6 that uses a 2 (2 + 4). Then add 8 + 2 to make 10 and finish by adding 4 to the 10.

Subtraction

- Use what you know about addition. Look at 11 – 6 and think, "What can I add to 6 to make a total of 11?"
- Think about subtraction as comparing two numbers. Think of how much more one number is than the other. For 10 – 7, think about how much more you need to count to get to 10.
- Think of subtraction as the difference between two numbers. Put the two numbers on a "pretend" number line. Think about the difference between those numbers.



- If ten is in between the numbers, think back down through 10. For 15 – 8, think 15 – 5 is ten, then take off 3 more to get to 7.

Multiplication

- Practice skip counting by every single digit number! Kids who follow football often know the x 7's without any effort!
- Look for ways to double. You know 2 x 8 = 16. 4 x 8 is the double of 2 x 8, so 4 x 8 will be 16 + 16.
- See the patterns. Multiples of 5 end in either a zero or a 5!
- Use the facts you know. If you know that 6 x 4 = 24, then you can reason that 7 x 4 is just one more group of 4.
- Think of the facts you know in another way. For 6 x 7, think about 3 x 7. 3 x 7 = 21 so 6 x 7 will be 21 + 21.
- Think 10! You know what 10 x 8 is. Use that to help you do 9 x 8 quickly. Think 10 x 8 and then subtract 8.

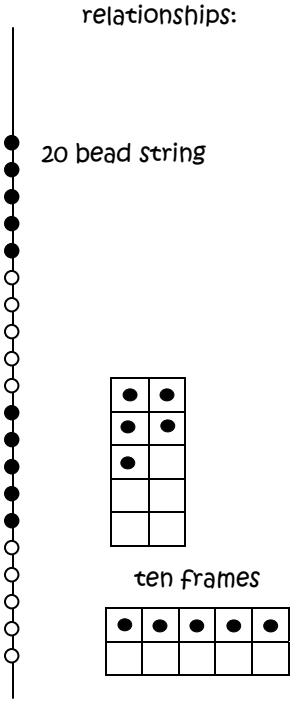
Division

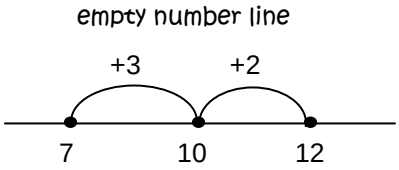
- Use the multiplication facts. For 32 ÷ 8, think what times 8 makes 32.
- Get as close as you can. For 54 ÷ 7, think about what you know that's close, perhaps 49 ÷ 7. Then adjust by adding or taking away another group of 7. 49 ÷ 7 will be 7 so 54 ÷ 7 will be 8!
- Think halves! For 64 ÷ 8, think 32 ÷ 4. Then double it.

Helping Students Develop Fact Fluency

Addition Facts

Powerful tools to visualize number relationships:





Before working on developing fluency, students need lots of practice with numbers. They need to count the objects used to model story problems. They need time to reflect on number relationships and discuss ways to use number relationships to compute. This practice will help them move from counting by ones to using part, part, whole relationships and combinations to make 10. Number relationships can be strengthened during number work by using a 20 bead string, ten frames, and empty number lines. These are powerful tools to help students visualize number relationships.

After lots of experiences in using objects to support counting, students begin to use their understanding of numbers to develop mental strategies for computing. Students develop these strategies in individual ways, but there does seem to be a general sequence of development shared by many students.

- Count on 1, 2, or 3 without using fingers or objects
- Learn about the zero property (0 + 8 = 8)
- Understand the commutative property (1 + 6 = 6 + 1)
- Recall doubles (5 + 5 = 10)
- Recognize part, part, whole relationships to make 10
- Use known doubles facts to reason about a computation close to a double (Because I know that 5 + 5 = 10, I think that 5 + 4 must be one less than 10. It must be 9.)
- Understand the value of working with, to and through 10

  by decomposing one addend (For 8 + 6, think of 6 as 2 + 4. Then 8 + 2 is 10. Add the 4 to make 14.)

  by compensating for a number close to 10 (For 9 + 7, pretend that 9 is 10, add the 7, and then correct the sum by taking away 1.)
- Use known facts to reason about a computation (For 7 + 5, I know 7 + 3 is 10, 7 + 4 is 11, so 7 + 5 is 12.)

Subtraction Facts



Graphs are powerful tools for developing concepts about comparing.

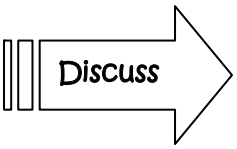
Subtraction is a challenging operation for primary grade students. They will need plenty of time to construct physical models to support their development of the number relationships involved in taking away. They need to broaden and deepen their understanding of subtraction to see it as comparing to find the difference as well as taking away. Ideas about comparing begin to develop when students learn to count on, usually in first grade. Ideas about finding the difference between two numbers can be fostered by analyzing bar graphs and later by using the empty number line. Students' concepts about differences and their ability to determine differences tend to become more evident in second grade. The following sequence describes a typical development of subtraction concepts and skills:

- Count back 1, 2, or 3 without using fingers
- Use part, part, whole relationships (I know 8 is made of 5 and 3. If I take away 5, I know I have 3 left.)
- Compare two numbers and count on to determine the difference
- Understand the value of working with, to and through 10 by decomposing the subtrahend (For 14 – 6, first work to 10 by taking away 4, then take away 2 to make 8.)
- Use known facts to reason about a computation (For 15 – 7, think I know 14 – 7 is 7 so 15 – 7 must be 8.)

Multiplication Facts

Students begin modeling multiplication story problems in kindergarten. They need to construct physical models to hold the sets in order to count them by ones. As students learn counting sequences such as counting by 2's or by 5's, they start to use those counting sequences to think about the total quantity of two or three groups. However, they will still need to model to count more than a few sets of any quantities. It is quite acceptable for primary students to work on multiplication by building fluency with their *count by* strategies.

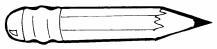
Developing Mental Computation Strategies



Begin working on developing mental computation strategies by asking students to share ideas about number relationships. Facilitate discussions that ask students to reflect on how they use number relationships to solve problems.

Ideas they might share include:

- Plus 1 just means going to the next number when you count.
- Minus 1 just means going back one number.
- You don't have to count when you add to 10. It is just that many past ten.
- When I add 6 + 5, I think about 5 + 5 and just add one more because 6 is one more than 5.
- It is easy to add 9. You just pretend that it is 10, add the rest, and then take one away, 'cuz you added 1 to make the 10.
- To add 9, just take 1 from the other number. That makes 10. Then just add the rest.
- I know that 7 + 4 is 11 so 7 + 5 has to be one more than 11.
- To subtract 8 from 14, I first take away 4 to get to 10 and then take away 4 more, to get to 6.
- I know that 2 times 6 is 12, so to get 4 times six, I just double 12, 'cuz 4 is the double of 2.

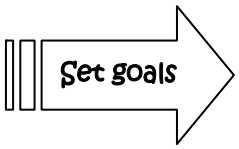


Write students' ideas for using number relationships on a chart posted in the classroom. Add to it as students think of other ideas. Expect students to explain their strategies.

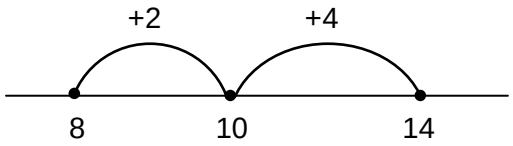


Allow students to work without time pressure until their number sense is very well developed. If students feel time pressure, they often revert to counting on or using fingers to track counting.

When students use well developed number sense strategies, teachers:



- help them identify the facts that they know and those that they still use counting strategies to compute
- ask them to set personal goals to find number sense strategies for unknown facts
- have them make fact cards for their unknown facts
  - keep the number of cards small and focused, 8 – 10 cards at a time
  - write the numbers horizontally, 6 + 8 = ____
  - create an empty number line representation on the back of each card



- provide short periods of instruction to develop the number relationships and strategies for those facts

Developing Fluency with Fact Computations

John A. Van de Walle, in *Teaching Student-Centered Mathematics* (page 94) states, "Fortunately, we know quite a bit about helping students develop fact mastery, and it has little to do with quantity of drill or drill techniques. Three components or steps to this end can be identified:

1. Help students develop a strong understanding of the operations and of number relationships.
2. Develop efficient strategies for fact retrieval.
3. Then provide practice in the use and selection of those strategies."

"Fortunately, we know quite a bit about helping students develop fact mastery, and it has little to do with quantity of drill or drill techniques."

-John A. Van de Walle

After students have developed strategies for using number relationships to compute basic facts, they are ready to work on developing fluency and automaticity. Card games, computer games, and worksheets using numbers that are at the student's independent mental calculation level (as determined by the Fact Interviews) are all ways to foster fluency. Short periods of practice sustained over weeks and months are effective in building proficiency in basic fact computations. If games have a competitive element, create groups of students who have similar fact proficiencies.

Fact fluency progress should be personal information. Instruction to support individual needs is more effective than posting students' progress on class charts.

Fact Fluency Targets K—5

Grade	Number Sense Development as Demonstrated By Fact Computation Standards See MMSD K-5 Grade Level Mathematics Standards
K	Not Applicable (See following grades if student demonstrates number sense at those levels)
1	**Demonstrate fluency with addition facts for:** • **'within' ten facts (sums less than 10)** • **combinations to make 10** Available Assessment: *Fact Interview A*—Sums Within and To 10
2	Demonstrate fluency with **all** addition facts: • **doubles** • **doubles ± 1** • **'across ten' facts (sums greater than ten)** • 'within' ten facts (sums less than 10) • combinations to make 10 Available Assessment: *Fact Interview B*—Sums to 20 Within, To and Across 10
3	**Demonstrate fluency using part-whole relationships, comparison, the concept of difference, or recall to determine the results of subtraction for facts.** Available Assessment: *Fact Interview C*—Differences Less Than, Equal To, and Greater Than 3 **Demonstrate fluency with multiplication facts for x2s, x4s, x5s, x3s.** (2, 5, 4, and 3 as multiplier or multiplicand) Available Assessment: *Fact Interview C*—2, 5, 4, and 3 as Multiplier or Multiplicand
4	**Demonstrate fluency with all multiplication facts (x9s, x6s, x7s, x8s**, x2s, x4s, x5s, x3s). (2, 5, 4, 3, **9, 6, 7, 8 as multiplier or multiplicand)** Available Assessment: *Fact Interview D*—All Multiplication Facts **Use the inverse relationship to determine the results of a division using multiplication facts.**
5	**Know division facts and the first ten multiples of 2, 3, 4, 5, 6, 7, 8, 9, 10, and 25.** Available Assessment: *Fact Interview E*—All Division Facts Use the inverse relationship to determine the results of a division using multiplication facts.

Bold face type indicates a new expectation at this grade level

Grading Guide Rubrics at the Proficient Level 2nd/3rd & 4th Quarter

Grade	Quarter(s)	Number Sense Development as Demonstrated By Fact Computation Standards
K		Not Applicable (See following grades if student demonstrates number sense at those levels)
1	2nd/3rd	*Fluently knows** addition facts for +1s, doubles to 5 + 5 Available Assessment: Fact Interview A—Sums Within and To 10 (First Two Columns)
	4th	*Fluently knows** addition facts for 'within' ten facts, and combinations to make 10 Available Assessment: Fact Interview A—Sums Within and To 10
2	2nd/3rd	*Fluently knows** addition facts for 'within' ten facts, combinations to make 10 and a few sums greater than ten. Available Assessment: Fact Interview B—Sums to 20 Within, To and Across 10 (First three columns)
	4th	*Fluently knows* *all* addition facts. Available Assessment: Fact Interview B—Sums to 20 Within, To and Across 10
3	2nd/3rd	*Fluently knows** differences of 1, 2, and 3 *Available Assessment*: Subtraction Fact Interview C Differences Less Than, Equal To, and Greater Than 3 (First Two Columns) *Fluently knows** multiplication facts with multiplier or multiplicand of 2 and 5 Available Assessment: Fact Interview C—2, 5, 4, and 3 as Multiplier or Multiplicand (First Three Columns)
	4th	*Uses part-whole relationships, comparison, the concept of difference, and recall to determine subtraction facts.* Available Assessment: Subtraction Fact Interview C— Differences Less Than, Equal To, and Greater Than 3 *Fluently knows** multiplication facts with multiplier or multiplicand of 2, 5, 4, and 3. Available Assessment: Fact Interview C—2, 5, 4, and 3 as Multiplier or Multiplicand
4	2nd/3rd	Knows *some* multiplication facts with multiplier or multiplicand of 6 – 9. Available Assessment: Fact Interview D—All Multiplication Facts (First Four Columns)
	4th	*Fluently knows** all multiplication facts. Available Assessment: Fact Interview D—All Multiplication Facts

Fluently knows means using knowledge of number size and operation, the child selects an efficient strategy (from a variety of known strategies).

Grading Guide Rubrics at the Proficient Level 2nd, 3rd & 4th Quarter (con't)

Grade	Quarter(s)	Number Sense Development as Demonstrated By Fact Computation Standards
5	2nd/3rd	Knows *some* division facts. Available Assessment: Fact Interview E—All Division Facts (First Two Columns) Identifies *some* multiples of 2-10, and 25. Determines *some* factors for numbers up to 100.
	4th	*Fluently knows* all division facts.* Available Assessment: Fact Interview E—All Division Facts Identifies multiples of 2-10 and 25. Determines factors of numbers up to 100.

*** *Fluently knows*** means the child uses his/her knowledge of number size and operation, when selecting an efficient computation strategy (from a variety of known strategies).

Problem Solving Interviews

There are three interviews.

1. *Interview K*
2. *Interview A – First Grade*
3. *Interview B – Second Grade*

Each interview has four elements:

1. **problem types** expected for proficiency at each grade level (Each grade level's problem types are indicated by bolding and underlining the problem type label.)
2. **range of number sets**, organized developmentally from those requiring the least amount of mental computation skills to those requiring the most flexibility in working with number (The target set of numbers for end of year proficiencies are in bold font and are underlined.)
3. **range of solution strategies** from the most concrete (direct modeling) to the most abstract (facts, standard algorithm) (The target solution strategies for end of year proficiencies are in bold font and are underlined.)
4. **space for making notes** of the student's explanations of how she/he solved the problem

Purposes:

- illuminate the problem types that a student can comprehend
- identify the number range that a student can compute mentally
- document the ways a student works with number to compute mentally
- determine the kinds of models a student creates when the number sizes exceed her/his mental number constructs
- communicate a student's progress to family and future teachers
- monitor a student's progress over time

Materials

- copy of the appropriate problem type interview
- one pencil for the teacher
- container of cubes organized in towers (trains) of 10
- scrap paper on which to write number sets (if wanted)

Conducting the interview:

The interview is used to determine the problem types and number sizes that a student can solve using mental computations. The interview is also used to determine the number size at which a student needs to create a model to count in order to solve the problem. A teacher may use the fact interview information to select a number size at which to begin the interview. Stay with the same problem type, using number sets with increasing number sizes until the student gives an inaccurate answer or indicates frustration in solving the problem. Then ask the student to solve or check that problem with those numbers by using the cubes. If the student solves the problem using cubes or shows that he/she does not comprehend the language of the problem type, move on to the next problem type.

- ☑ Find a place where you and the student can sit next to each other.
- ☑ Have a copy of the interview in front of you.

- ☑ Say: ***"I am going to tell you some story problems. Do you want to have your name in them or should we use someone else's name?"***
- ☑ Say: ***"I want you to use your great thinking skills to figure out the answer to the questions and tell me your thinking steps. Sometimes I am going to ask you to check your answer by using cubes or to use cubes to help you figure out the answer."***
- ☑ Tell the *join, result unknown* story problem, selecting a set of numbers as a starting point. Use alternative wording and name choices if it fosters comprehension.
- ☑ Write a 1 on the line in front of the set of numbers used first. (You may want to write the set of numbers on a piece of scrap paper so as not to tax the student's short term memory.)
- ☑ Write the student's answer after the set of numbers. If inaccurate, make a short horizontal line over the number you wrote.
- ☑ Write a 1 on the line in front of the strategy that the student used to solve the problem with the first set of numbers.
- ☑ If you do not understand the thinking steps a student uses, ask him/her to explain how he/she solved the problem. Make notes to indicate the ways a student works with the given numbers.
- ☑ Continue with the same problem type, using another set of numbers. You will have to decide whether to continue with the very next set or skip a set or two if you feel that student can use mental calculations for those sets of numbers.
- ☑ Write a 2 on the line in front of the second set of numbers assessed. Write a 2 on the line in front of the strategy the student used to solve the problem using the second set of numbers.
- ☑ Continue giving more challenging number sets within the same problem type until the student can't solve the problem mentally, gives an inaccurate answer, or shows other signs of frustration.
- ☑ When the student gives an inaccurate answer or indicates frustration, ask him/her to use cubes to check or to help solve the problem.
- ☑ Move to the next problem type and repeat the process.

- ☑ Proceed with the interview until you have assessed solution strategies for each of the problem types expected at the student's grade level.
- ☑ If a student can solve all the problem types using mental computation strategies, you may want to try the next problem solving interview.

Problem Solving Interview K Name _____ Date _____ Recorder _____

(End of Year Grade Level Targets are in Bold)

JRU _____ had ____ marbles. Sergio gave him/her ____ more. How many marbles did _____ have in all?	___ (3, 1) ___ (1, 4) ___ (5, 2) ___ (14, 1) ___ (6, 3) ___ (3, 12) ___ (5, 5) ___ **(10, 6)** ___ (18, 10) ___	_____ **Direct modeling by 1's,** 10's _____ Counting on by 1's _____ Facts, derived / recalled Algorithm(s): _____ incrementing _____ compensating _____ 10's, 1's _____ standard
SRU _____ had ____ marbles. He/she gave ____ to a friend. How many marbles did _____ have left?	___ (3, 1) ___ (5, 2) ___ (16, 1) ___ (13, 3) ___ (8, 5) ___ (10, 4) ___ (9, 8) ___ (13, 9) ___ **(16, 10)** ___	_____ **Direct modeling by 1's,** 10's _____ Counting on/back by 1's _____ Facts, derived, recalled Algorithm(s): _____ incrementing _____ compensating _____ 10's, 1's _____ standard
M _____ has ____ bags of crackers. There is/are ____ cracker(s) in every bag. How many crackers does _____ have in all?	___ (4, 1) ___ (3, 2) ___ (3, 5) ___ (2, 10) ___ (4, 4) ___ **(5, 3)** ___	_____ **Direct modeling** _____ Counting on by 2's, 5's, 10's _____ Repeated addition _____ Skip counting _____ Doubling _____ Facts, derived / recalled _____ Algorithm(s)
MD _____ has ____ crackers on a plate. He/she has to put them all away in little bags. He/she can fit ____ cracker(s) in every bag. How many bags will he/she use in order to put away all of the crackers?	___ (3, 1) ___ (6, 2) ___ (15, 5) ___ (12, 3) ___ **(18, 6)** ___	_____ **Direct modeling** _____ Counting back _____ Repeated subtraction _____ Repeated addition _____ Skip counting _____ Facts, derived / recalled _____ Algorithm(s)
JCU _____ has ____ marbles. How many more marbles does he/she need to have ____ marbles altogether? (Alternative wording: He/she wants _____. Or He/she needs to have ____. How many more does he/she need?)	___ (3, 4) ___ (5, 7) ___ (6, 9) ___ (20, 26) ___ (3, 10) ___ (9, 17) ___ (58, 68) ___ (48, 60) ___ (37, 53) ___	_____ Direct modeling by 1's, 10's _____ Counting on by 1's, _____ Facts, derived / recalled Algorithm(s): _____ incrementing _____ compensating _____ 10's, 1's _____ standard

Problem Solving Interview K Name _____ Date _____ Recorder _____
(End of Year Grade Level Targets are in Bold)

CDU _____ has ____ marbles. Sergio has ____ marbles. How many more marbles does _____ have than Sergio? (Alternative wording: Who has more marbles? How many more?)	___ (5, 2) ___ (7, 4) ___ (10, 6) ___ (46, 40) ___ (8, 3) ___ (13, 6) ___ (60, 50) ___ (65, 45) ___ (70, 55) ___ (74, 38) ___	_____ Direct modeling by 1's, 10's _____ Counting on/back by 1's _____ Facts, derived / recalled Algorithm(s) _____ incrementing _____ compensating _____ 10's, 1's _____ standard
PPW, WU _____ had ____ red marbles and ____ blue marbles. How many marbles did _____ have in all?	___ (7, 1) ___ (3, 5) ___ (13, 2) ___ (10, 8) ___ (8, 4) ___ (6, 7) ___ (27, 20) ___ (39, 55) ___	_____ Direct modeling by 1's, 10's _____ Counting on/back by 1's _____ Facts, derived / recalled Algorithm(s): _____ incrementing _____ compensating _____ 10's, 1's _____ standard
SCU _____ had ____ marbles. He/she gave some to a friend. Now he/she has ____ left. How many marbles did _____ give to his/her friend?	___ (6, 5) ___ (9, 7) ___ (15, 12) ___ (9, 1) ___ (10, 4) ___ (15, 5) ___ (17, 9) ___ (84, 74) ___ (61, 29) ___	_____ Direct modeling by 1's, 10's _____ Counting on/back _____ Difference, Facts Algorithm(s): _____ incrementing _____ compensating _____ 10's, 1's _____ standard
PD _____ has ____ crackers on a plate. He/she has to put them away in ____ little bags. He/she wants to put the same number in each bag. How many crackers will he/she put in each bag?	___ (6, 2) ___ (10, 5) ___ (12, 3) ___ (16, 4) ___ (18, 3) ___	_____ Direct modeling _____ Counting back _____ Repeated subtraction _____ Repeated addition _____ Skip counting _____ Facts, derived / recalled _____ Algorithm(s)
PD with a fraction in answer There is/are ____ brownie(s) on the plate. ____ friends are going to share them. Each friend will get the same amount. How much brownie will each friend get?	___ **(1, 2)** ___ (3, 2) ___ (1, 4) ___ (5, 4) ___ (7, 3) ___	_____ **Direct modeling** _____ Counting back _____ Repeated subtraction _____ Repeated addition _____ Skip counting _____ Facts, derived / recalled _____ Algorithm(s)

Problem Solving Interview A Name _____ Date _____ Recorder _____
(End of Year Grade Level Targets are in Bold)

JRU
_____ had ____ marbles.
Sergio gave him/her ____ more.
How many marbles did _____ have in all?

___ (5, 1)
___ (47, 2)
___ (3, 7)
___ **(10, 6)**
___ (9, 4)
___ (5, 7)
___ (34, 10)
___ (20, 48)
___ (36, 57)
___ (89, 63)

_____ Direct modeling by 1's, 10's
_____ Counting on by 1's
_____ **Facts, derived / recalled**
Algorithm(s):
_____ incrementing
_____ compensating
_____ 10's, 1's
_____ standard

SRU
_____ had ____ marbles.
He/she gave ____ to a friend.
How many marbles did _____ have left?

___ (6, 1)
___ (10, 2)
___ **(12, 3)**
___ (10, 8)
___ (8, 5)
___ (16, 6)
___ (12, 7)
___ (42, 10)
___ (54, 19)
___

_____ Direct modeling by 1's, 10's
_____ **Counting on/back by 1's**
_____ Facts, derived / recalled
Algorithm(s):
_____ incrementing
_____ compensating
_____ 10's, 1's
_____ standard

M
_____ has ____ bags of crackers.
There are ____ crackers in every bag.
How many crackers does _____ have in all?

___ (2, 5)
___ (3, 2)
___ (4, 5)
___ **(4, 10)**
___ (4, 4)
___ (3, 8)
___ (9, 4)
___

_____ Direct modeling
_____ **Counting on by 2's, 5's, 10's**
_____ Repeated addition
_____ Skip counting
_____ Doubling
_____ Facts, derived / recalled
_____ Algorithm(s)

MD
_____ has ____ crackers (on a plate).
He/she wants to put them in little bags.
He/she can put ____ crackers in every bag.
How many bags will he/she need/use
(to put away all of the crackers?)

___ (6, 2)
___ (15, 5)
___ **(30, 10)**
___ (16, 4)
___ (18, 3)
___ (24, 6)
___

_____ **Direct modeling**
_____ Counting back
_____ Repeated subtraction
_____ Repeated addition
_____ Skip counting
_____ Facts, derived / recalled
_____ Algorithm(s)

JCU
_____ has ____ marbles.
How many more marbles does he/she need to have ____ marbles altogether?

(Alternative wording:
He/she wants _____.
Or He/she needs to have ____.
How many more does he/she need?)

___ (3, 4)
___ (5, 7)
___ (8, 11)
___ **(20, 26)**
___ (3, 10)
___ (9, 17)
___ (8, 14)
___ (58, 68)
___ (48, 60)
___ (37, 53)
___

_____ Direct modeling by 1's, 10's
_____ **Counting on by 1's**
_____ Facts, derived / recalled
Algorithm(s):
_____ incrementing
_____ compensating
_____ 10's, 1's
_____ standard

Problem Solving Interview A Name _____ Date _____ Recorder _____
(End of Year Grade Level Targets are in Bold)

CDU
_____ has ____ marbles.
Sergio has ____ marbles.
How many more marbles
does _____ have than Sergio?
(Alternative wording:
Who has more marbles? How many more?)

___ (5, 2)
___ (7, 4)
___ (10, 6)
___ **(46, 40)**
___ (8, 3)
___ (13, 6)
___ (60, 50)
___ (65, 45)
___ (70, 55)
___

_____ Direct modeling by 1's, 10's
_____ Counting on by 1's
_____ **Facts, derived / recalled**
Algorithm(s):
_____ incrementing
_____ compensating
_____ 10's, 1's
_____ standard

PPW, WU
_____ had ____ red marbles
and ____ blue marbles.
How many marbles did _____ have in all?

___ (7, 1)
___ (2, 35)
___ (3, 6)
___ **(10, 8)**
___ (8, 4)
___ (6, 7)
___ (27, 10)
___ (20, 53)
___ (39, 45)
___

_____ Direct modeling by 1's**,** 10's
_____ Counting on by 1's
_____ **Facts, derived / recalled**
Algorithm(s):
_____ incrementing
_____ compensating
_____ 10's, 1's
_____ standard

SCU
_____ had ____ marbles.
He/she gave some to a friend.
Now he/she has ____ left.
How many marbles did _____
give to his/her friend?

___ (6, 5)
___ (9, 7)
___ **(15, 12)**
___ (9, 1)
___ (10, 4)
___ (15, 5)
___ (17, 9)
___ (84, 74)
___ (56, 36)
___ (61, 29)
___

_____ Direct modeling by 1's**,** 10's
_____ **Counting on/back by 1's**
_____ Facts, derived / recalled
_____ Algorithm(s):
_____ incrementing
_____ compensating
_____ 10's, 1's
_____ standard

PD
_____ has ____ crackers on a plate.
He/she has to put them away
in ____ little bags. He/she wants to put
the same number in each bag.
How many crackers will he/she put
in each bag?

___ (6, 2)
___ (10, 5)
___ **(12, 3)**
___ (16, 4)
___ (18, 3)
___

_____ **Direct modeling**
_____ Counting back
_____ Repeated subtraction
_____ Repeated addition
_____ Skip counting
_____ Facts, derived / recalled
_____ Algorithm(s)

PD with a fraction in answer
There is/are ____ brownie(s) on the plate.
____ friends are going to share them.
Each friend will get the same amount.
How much brownie will each friend get?

___ (1, 2)
___ (3, 2)
___ **(1, 4)**
___ (5, 4)
___ (7, 3)
___

_____ **Direct modeling**
_____ Counting back
_____ Repeated subtraction
_____ Repeated addition
_____ Skip counting
_____ Facts, derived / recalled
_____ Algorithm(s)

Problem Solving Interview B Name _____ Date _____ Recorder _____
(End of Year Grade Level Targets are in Bold)

JRU _____ had ____ marbles. Sergio gave him/her ____ more. How many marbles did _____ have in all? (At this level, students often solve **PPW, WU** problems in the same ways that they solve JRU problems.)	___ (10, 6) ___ (4, 9) ___ (5, 7) ___ (34, 10) ___ (20, 45) ___ (23, 38) ___ **(25, 75)** ___ (89, 63) ___ (264, 128) ___	_____ Direct modeling by 1's, 10's _____ Counting on by 1's _____ Facts, derived / recalled **Algorithm(s):** _____ incrementing _____ compensating _____ 10's, 1's _____ standard
SRU _____ had ____ marbles. He/she gave ____ to a friend. How many marbles did _____ have left?	___ (10, 8) ___ (8, 5) ___ (14, 6) ___ (12, 7) ___ (67, 7) ___ (42, 10) ___ (68, 35) ___ **(54, 19)** ___ (197, 135) ___	_____ Direct modeling by 1's, 10's _____ Counting back,/on by 1's _____ Facts, derived / recalled (Difference and take away) **Algorithm(s):** _____ incrementing _____ compensating _____ 10's, 1's _____ standard
M _____ has ____ bags of crackers. There are ____ crackers in each bag. How many crackers does _____ have in all?	___ (7, 2) ___ (6, 5) ___ (4, 10) ___ (5, 3) ___ **(3, 8)** ___ (4, 4) ___ (9, 4) ___	_____ Direct modeling _____ **Counting on by 2's, 5's, 10's, 4's, 3's** _____ Repeated addition _____ Skip counting _____ Doubling _____ Facts, derived / recalled _____ Algorithm(s)
MD _____ has ____ crackers (on a plate). He/she has to put them in little bags. He/she can put ____ crackers in each bag. How many bags will he/she use in order to put away all of the crackers?	___ (15, 3) ___ (30, 10) ___ (40, 5) ___ (24, 4) ___ (18, 3) ___ **(24, 6)** ___	_____ **Direct modeling** _____ Counting back _____ Repeated subtraction _____ Repeated addition _____ Skip counting _____ Facts, derived / recalled _____ Algorithm(s)
JCU _____ has ____ marbles. How many more marbles does he/she need to have ____ marbles altogether? (Alternative wording: He/she wants (to have) _____. How many more does he/she need (to get)?)	___ (8, 11) ___ (20, 26) ___ (3, 10) ___ (9, 17) ___ (15, 22) ___ (58, 68) ___ (48, 60) ___ (37, 53) ___ **(65, 100)** ___ (75, 125) ___ (348, 452) ___	_____ Direct modeling by 1's, 10's _____ Counting on/back by 1's _____ Facts, derived / recalled **Algorithm(s):** _____ incrementing _____ compensating _____ 10's, 1's _____ standard

Problem Solving Interview B Name _____ Date _____ Recorder _____
(End of Year Grade Level Targets are in Bold)

CDU _____ has ____ marbles. Sergio has ____ marbles. How many more marbles does _____ have than Sergio? (Alternative wording: Who has more marbles? How many more?)	___ (10, 6) ___ (46, 40) ___ (8, 3) ___ (13, 6) ___ (60, 50) ___ (65, 45) ___ (70, 55) ___ (81, 47) ___ **(100, 48)** ___ (125, 50) ___ (453, 374) ___	_____ Direct modeling by 1's, 10's _____ Counting on/back by 1's _____ Facts, derived / recalled **Algorithm(s):** _____ incrementing _____ compensating _____ 10's, 1's _____ standard
SCU _____ had ____ marbles. He/she gave some to a friend. Now he/she has ____ left. How many marbles did _____ give to his/her friend?	___ (15, 12) ___ (10, 4) ___ (15, 5) ___ (17, 9) ___ (84, 74) ___ (56, 36) ___ **(61, 29)** ___ (282, 136) ___	_____ Direct modeling by 1's, 10's _____ Counting on/back by 1's _____ Facts, derived / recalled **Algorithm(s):** _____ incrementing _____ compensating _____ 10's, 1's _____ standard
PPW,PU _____ has ____ marbles. ____ of his marbles are blue. The rest of his marbles are green. How many green marbles did _____ have?	___ (57, 50) ___ (9, 6) ___ (15, 9) ___ (90, 50) ___ (60, 38) ___ **(71, 36)** ___ (483, 275) ___	_____ Direct modeling by 1's, 10's _____ Counting on/back by 1's _____ Facts, derived / recalled **Algorithm(s):** _____ incrementing _____ compensating _____ 10's, 1's _____ standard
PD _____ had ____ crackers (on a plate). He/she put them away in ____ little bags. He/she put the same number in each bag. How many crackers did he/she put in each bag?	___ (18, 2) ___ (30, 5) ___ (16, 4) ___ (30, 10) ___ (18, 3) ___ (24, 8) ___ **(30, 6)** ___	_____ **Direct modeling** _____ Counting back _____ Repeated subtraction _____ Repeated addition _____ Skip counting _____ Facts, derived / recalled _____ Algorithm(s)
PD with a fraction in answer There is/are ____ brownie(s) on the plate. ____ friends are going to share them. Each friend will get the same amount. How much brownie will each friend get?	___ (1, 4) ___ (5, 2) ___ (9, 4) ___ (1, 8) ___ (6, 4) ___ (4, 3) ___ (10, 3) ___ (13, 6) ___ **(18, 8)** ___	_____ **Direct modeling** _____ Counting back _____ Repeated subtraction _____ Repeated addition _____ Facts, derived / recalled _____ Algorithm(s)

Problem Solving Interview B Name _____ Date _____ Recorder _____
(End of Year Grade Level Targets are in Bold)

M with fraction as operator There are ____ balloons in a bag. You can have ____ of the balloons. How many balloons can you take?	___ (8, 1/2) ___ (20, 1/4) ___ (12, 1/3) ___ (16, 1/8) ___ **(6, 1/6)** ___	_____ **Direct modeling** _____ Facts, derived / recalled _____ Algorithm(s)
JSU _____ had some marbles. A friend gave him/her ____ marbles. Now he/she has ____ marbles. How many marbles did _____ have at the beginning? (Alternative wording: How many marbles did _____ have before his/her friend gave him/her ____ marbles?)	___ (6, 11) ___ (10, 18) ___ (8, 14) ___ (7, 16) ___ (50, 73) ___ (48, 91) ___ (76, 150) ___ (350, 500) ___	_____ Direct modeling by 1's, 10's _____ Counting back by 1's _____ Facts, derived / recalled Algorithm(s): _____ incrementing _____ compensating _____ 10's, 1's _____ standard
SSU _____ had some marbles. He/she gave ____ marbles to a friend. Now he/she has ____ marbles. How many marbles did _____ have at the beginning? (Alternative wording: How many marbles did _____ have before he/she gave ____ marbles to a friend?)	___ (3, 4) ___ (7, 5) ___ (5, 9) ___ (43, 10) ___ (25, 75) ___ (48, 37) ___ (542, 332) ___	_____ Direct modeling by 1's, 10's _____ Counting on by 1's _____ Facts, derived / recalled Algorithm(s): _____ incrementing _____ compensating _____ 10's, 1's _____ standard

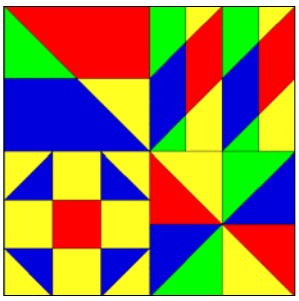
For more information

Glanfield, F., Bush, William S. & Stenmark, J. K. (Eds.). (2003). *Mathematics assessment: A practical handbook K-2*. Reston, VA: National Council of Teachers of Mathematics.

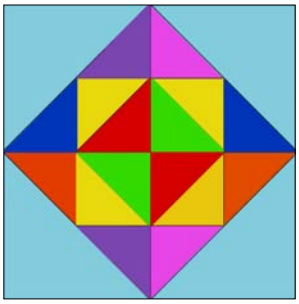
Sutton, J. & Krueger, A. (Eds.). (2002). *EDThoughts: What we know about mathematics teaching and learning*. Aurora, CO: Mid-continent Research for Education and Learning.

Van de Walle, J. A. (2001). *Elementary and middle school mathematics*. New York, NY: Longman.

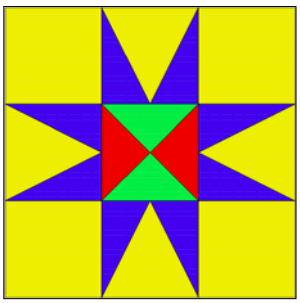
Van de Walle, J. A. & Lovin, L. H. (2006). *Teaching student-centered mathematics: Grades K-3*. Boston, MA: Allyn and Bacon Pearson Education, Inc.



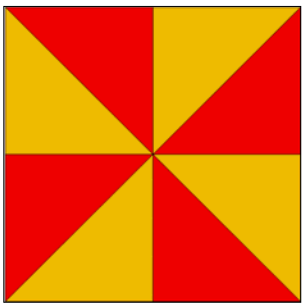
PROBLEM SOLVING



NUMBER WORK



INSPECTING EQUATIONS



FLUENCY & MAINTENANCE

Chapter 5

Organizing for Instruction

Organizing For Instruction

Teachers in the primary grades have to plan for *everything* that impacts instruction. They consider such things as student interests, room arrangements and daily schedules as they make plans for each day's mathematics instruction. Some things are more critical than others for impacting student success.

Two critical things to plan for are:

1. goals and activities most effective for learning the mathematical content
2. each student's needs based on assessments

Effective daily instruction also depends on planning for materials, groupings and allocation of time. When planning, teachers consider the following:

- **manipulatives** and other materials needed and ways to organize the materials so students can access them
- **organization** of the kinds and sizes of groups and the ways students will move from group to group
- **time** allocations for focusing on *Problem Solving, Number Work, Inspecting Equations and Fluency & Maintenance*



Manipulatives for Instruction in the Primary Grades

Having manipulatives on hand to model problems with numbers and shapes and to measure objects is critical for building students' understandings during primary math instruction.

Manipulatives are a necessity for math instruction at all grade levels, but they are crucial for math instruction in the primary grades.

As schools acquire manipulatives, it is important to understand that the manipulatives used on a daily basis must be available in each classroom. Other manipulatives, such as balances or class sets of measuring tools, are used every now and then. Schools might want to store these in a central place to be shared, as classrooms need them. The following recommendations are based on a class size of 15 students.

Manipulatives needed for Number, Operations and Algebraic relationships include:

- hundred chart (pocket chart)
- 100 and 200 number charts
- wide variety of counters (teddy bears, frogs, jewels, dinosaurs, vehicles, etc.)
- two-color counters
- buttons—500 per classroom
- Unifix cubes
  - K – 60 per student
  - 1 – 120 per student
  - 2 – 180 per student
- 20 bead string
- dice with dots and dice with numerals
- arrow cards – 1 teacher set per classroom
  1 student set for every 2 students
- place value blocks (transparent and interlocking)
  - K - a starter set per classroom
  - 1 – 2
    - 50 ones (units) per student
    - 20 tens (rods) per student
    - 10 hundreds (flats) per student
    - several thousand (cubes)
- coins and dollar bills
- calculators



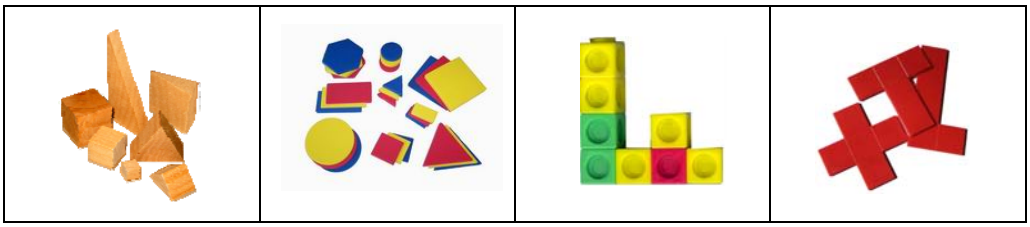
Working with arrow cards.

Manipulatives needed for Geometry include:



Working with Polydrons

- pattern blocks (4-5 buckets per classroom)
- Geoblocks (2-3 sets per classroom)
- geometric solids (4-5 sets of 12 solids per classroom)
- Attribute blocks (at least 1 set per classroom)
- Multi-links (1000 per classroom)
- Polydrons (at least 2 sets per classroom)
- 1 inch square tiles (1 bucket per classroom)
- 1 inch wooden cubes (2 - 3 buckets per classroom)
- mirrors (4" by 6") (at least 1 for every two students)
- geoboards and bands (at least 1 for every two students)
- tangrams (1 set per student)
- pentominoes (1 set per student)



Geoblocks | Attribute blocks | Multi-links | Pentominoes

Manipulatives needed for Geometry include:

- A variety of items to use as non-standard units, such as
  - measuring worms
  - links
  - 5 ounce paper cups
  - toothpicks
  - teddy bear counters
  - washers (quantities of several sizes)
- demonstration clock (geared)
- primary bucket balances (1 for every two students)
- funnels
- tools with standard units marked to measure length, capacity, and weight (rulers, yard and meter sticks, quart, pint and cup pitchers) ounce and gram weights

Storage of manipulatives

Teachers organize manipulatives in ways that:

- allow for students' independent access and use
- bring focus to mathematical ideas

Organizing to allow for students' independent access and use

- Communicate clearly how students should get and put away manipulatives
  - Does each child get the needed things or is there a person whose job it is to pass out and collect manipulatives?
  - When are students allowed or expected to get manipulatives?
  - How many students are able to share a specific manipulative?
- Store manipulatives in ways that allow for individual or small group use.
  - Keep Unifix cubes in small baskets so each student can efficiently access the cubes.
  - Keep a variety of pattern blocks in small containers or plastic bags so individuals or small groups can each have one container or bag.
- Label the locations for storing each kind of manipulative.
- Store the manipulatives that support the current learning activities in easily accessible places. Keep other manipulatives stored out of reach.
- Students should clearly understand which manipulatives they are allowed to use.





Organizing to bring focus to mathematical ideas:

- Store Unifix cubes in towers of 10, with 5 of one color and 5 of another. Having cubes organized in this way highlights the 5 and the 10. Students reference 5 to select 4 or 7 cubes. They reference 10 to select 9 or 8.



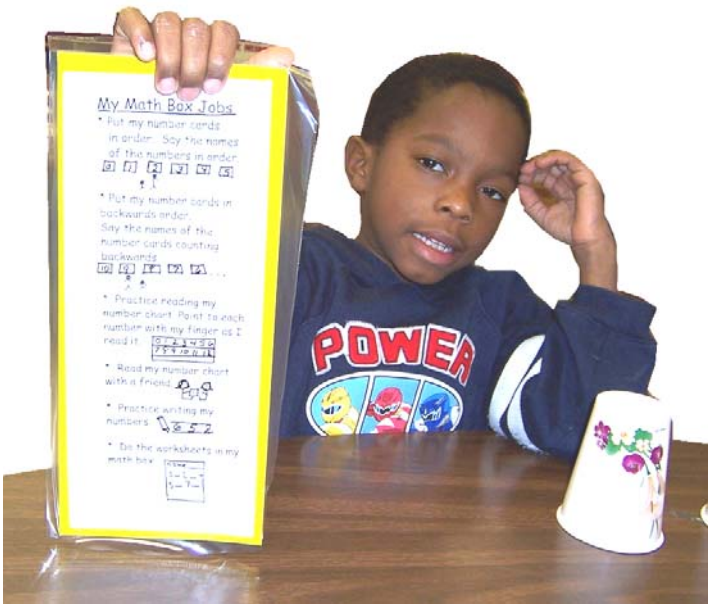
- Label manipulatives with both words and images. This reinforces the vocabulary of mathematics. *(See labels in Appendix.)*
- Put all of the geometry manipulatives together and label the storage area **Geometry**. Do the same with the manipulatives for **Measurement** and for **Number**. This structure indicates connections between tools and reinforces academic mathematical vocabulary. *(See labels in Appendix.)*

Structures that Foster Individual, Small, and Large Group Work

Primary students thrive on routines.

Students in the primary grades thrive on routine. Routines allow them to focus on the content of the activity instead of on behavioral and social expectations.

- Plan a sequence to the math hour so students can anticipate activities and expectations.
- Develop ways for students to transition from large group to small group to independent activities.
- Have clear expectations for how each area of the room is used during the math hour.
- Organize individual work so students can independently access it and hand it in. Many teachers place individual work in pocket folders or math boxes.
- Use a voice that indicates which group you are addressing – a louder voice for the large group's attention, a quieter voice for a small group's attention, and a whisper for an individual's attention.



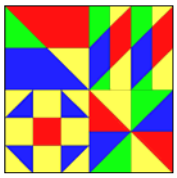
Time Allocation for the Four Block Math Hour

Students grow in confidence as they feel more and more competent. In the primary grades, students:

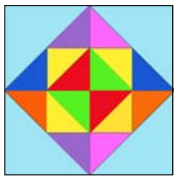
- begin by relying on their informal intuitive understandings to help them make sense of new problem situations
- stretch their vocabulary to learn to reflect on their work and share their thinking
- become aware of patterns when they work with numbers and shapes
- learn new math symbols and begin to understand that there are conventions in the ways those symbols are used
- use their strong sense of inquiry to begin making generalizations about number properties and operations

At least one hour of every school day is used for math instruction.

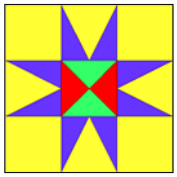
To have time to build this competence and confidence, math instruction easily takes an hour every day. This hour is filled with active, engaging experiences that focus students' attention on learning important math concepts and conventions. It takes repeated experiences to gain solid understandings of new mathematical concepts, conventions and skills. These experiences are guided by a teacher's careful posing of engaging problems and students' reflections on their solutions. During the hour, the students also have some time to practice their skills and build fluency.



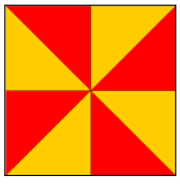
PROBLEM-SOLVING



NUMBER WORK



INSPECTING EQUATIONS



FLUENCY & MAINTENANCE

Organizing the Math Hour

The math hour is organized to keep the focus on important mathematical activities. The Four Blocks are: **problem solving, number work, inspecting equations, and fluency & maintenance.**

- **Problem solving** is the heart of math instruction. Problems may have a context and be set forth within a story. Problems can also be posed about ways to fill shapes or construct three-dimensional objects.
- Thinking about **numbers** themselves, without a story context, is also an important element of math instruction.
- Students in kindergarten, first, and second grade develop concepts about equality. During **inspecting equations** activities, they do this by analyzing the ways in which equations communicate equality relationships.
- When students are able to use their new knowledge and skills independently, they need practice in order to develop fluency. The fourth block of instruction focuses on independent mathematical work. This block is called **fluency & maintenance**.

The diagram below shows the four blocks and recommends the amount time for each block of the math hour at each grade level.

The lines within the diagram are dotted to indicate that the activities in each block can easily cross over into the other blocks. ***Learning activities in each block support the learning activities in the other blocks.*** The blocks can be organized in any sequence that best meets student needs and the class schedule.

Instruction	**Problem Solving** K - 25 minutes 1 - 20 minutes 2 - 25 minutes	**Number Work** K - 25 minutes 1 - 20 minutes 2 - 15 minutes	**Inspecting Equations** (about 5 minutes of the hour, less for K)
Practice	**Fluency & Maintenance** K—as needed, 1—about 15 minutes, 2—about 15 minutes		

Kindergarten

In kindergarten, so much is new in number work and problem solving. There is just a little introductory work on concepts of equality. Less time is needed for fluency and maintenance at the kindergarten level because so many of the activities are new and need a teacher's guidance. In kindergarten, the math hour is almost evenly divided between problem solving and number work, 25 minutes for problem solving, 25 minutes for number work. Small amounts of time are spent learning about the equal sign during the inspecting equations block.



The time allocation for each block changes from grade to grade.

First Grade

In first grade, the time allocation for each block changes to have more time for developing fluency and inspecting equations. First graders are introduced to many conventions, such as the symbolic notations for multi-digit numbers, operations, and statements of equality. Students need time during inspecting equations activities to analyze the conventions of symbolic representations and to consider the properties of numbers and operations. Five minutes a day to think about the information and concepts represented in equations provides consistent experience that fosters strong conceptual development. First graders also need time to build fluency with their counting strategies and number relationships for addition facts to ten. They develop flexible and fluent mathematical thinking through daily practice solving problems with numbers at their independent mental computation levels.

Second Grade
In second grade, one of the challenges is to develop an understanding of place values. Provide about 15 minutes of number work a day to concentrate on activities to develop number relationships and place value concepts. Second graders need a little more time for problem solving to develop strategies for working with place value when computing. By second grade, students have learned many conventions, concepts and vocabulary. This knowledge needs to be revisited again and again during the fluency and maintenance block in order to continue creating strong connections.

Two Teachers' Ways of Organizing the 4 Blocks

Each teacher will organize the math hour in ways that are most effective for her/his students and teaching situation.

First Grade Teacher #1: This teacher begins the math hour by engaging the whole group in inspecting equations activities. The students then move to the fluency & maintenance independent activities in their math folders or math boxes. While students are working independently, the teacher meets with small groups of students to guide their development of problem solving strategies. This teacher brings the whole class together again to do a number work activity. Each day there is a different number work routine. Sometimes students lead the class in number work routines. This teacher provides instruction in all four blocks every day.

First Grade Teacher #2:
Four days of the week, this teacher begins the day with two number work activities. After introducing the second activity, the students work in small groups to complete the task. When the students complete the number work activity, they get their independent fluency & maintenance activities. While students work independently, the teacher meets with small groups for problem solving instruction. The teacher brings the whole class together to play a true/false number sentence game for inspecting equations. On the fifth day of the week, the students engage in fact practice activities. Four days of the week, this teacher provides instruction in three blocks (problem solving, number work, inspecting equations). The class spends the fifth day working in the fluency & maintenance block.

Advantages of Organizing the Math Hour into 4 Blocks

Structuring the math hour in four blocks

- keeps the focus on important mathematical content and processes
- facilitates using appropriate settings to develop students'
  - **competency (ability)**
  - **proficiency (skill)**
  - **fluency (ease)**
- provides for ways to have time to differentiate for small group or individual needs based on assessment data
- provides opportunities for students to examine the same concepts within a variety of learning contexts

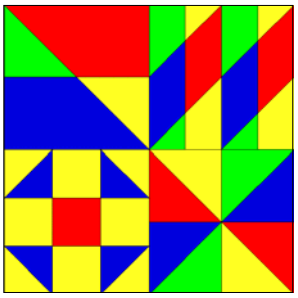


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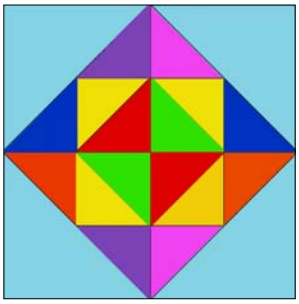
Bachman, V. (2003). *First-grade math: A month-to-month guide*. Sausalito, CA: Math Solutions Publications.

Litton, N. (2003). *Second-grade math: A month-to-month guide*. Sausalito, CA: Math Solutions Publications.

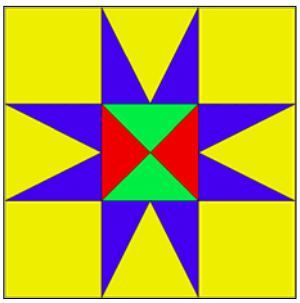
Richardson, K. (2002). *Math time: The learning environment*. Peabody, MA: Educational Enrichment, Inc. Didax Educational Resources.



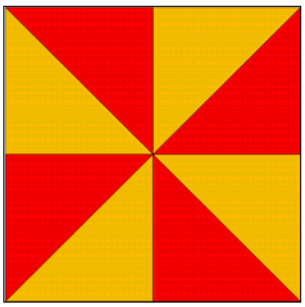
PROBLEM SOLVING



NUMBER WORK



INSPECTING EQUATIONS

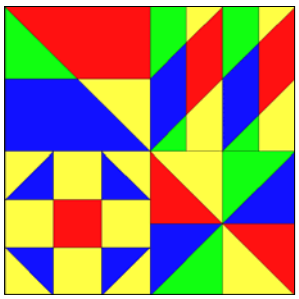


FLUENCY & MAINTENANCE

Chapter 6

Problem Solving

Learning activities in each block
support the learning activities in the other blocks.



Problem Solving
20-25 minutes

Problem Solving Block

Problem solving activities challenge students to solve problems from all the content strands: number, operations and algebraic relationships; geometry; measurement; and data analysis and probability. During problem solving activities, students:

- identify and understand the elements of the problem and their relationships
- engage in finding one or more ways to answer the question(s) in the problem
- represent and share their strategy(ies)
- reason about the accuracy of their solutions
- compare their strategy(ies) with the strategies used by classmates

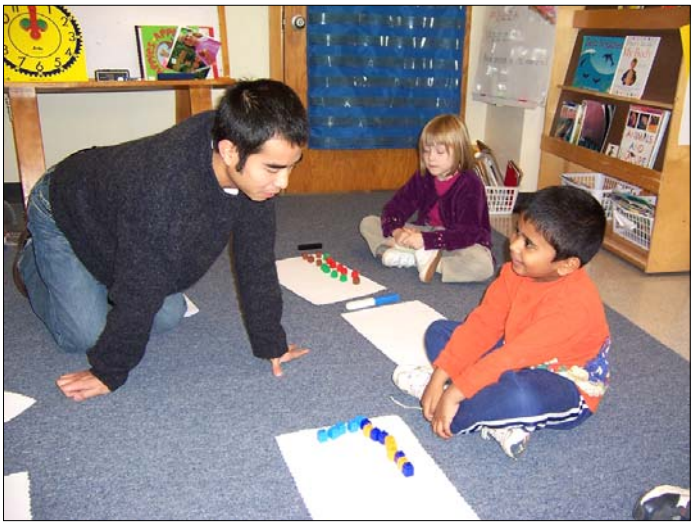
Answers to the questions in the problems are not necessarily obvious nor are the solution strategies needed to arrive at the answers. In kindergarten, the context of the problems offers support in thinking about what strategies to use. The verbs in the problem indicate the operations to use. Consider this problem: *"J'Quan had five rocks. He found 7 more rocks."* A beginning problem solver recognizes the meaning of the word *found* and knows what to do with the 7 more rocks.

As students develop their understanding about number relationships, the context of the problem becomes less important in providing clues for solution strategies. Consider this problem: *"J'Quan lost five rocks on his way to school. On the way home, he lost 7 rocks. How many rocks did he lose that day?"* A beginning problem solver might be confused about what to do because of the word *lost*, thinking that maybe this was a take-away problem. However, a more experienced problem solver would pay less attention to the meaning of the word *lost* and more attention to the broader context and the relationships of the numbers in the problem. As students build deeper and more complex knowledge about numbers and operations, they are able to solve problems in more than one way.

Depending upon the purpose of the problem solving tasks (to foster growth in a specific strategy, to expand students' understanding of a new set of numbers, etc.) a teacher plans for working with a large group, with a small group, or with an individual during problem solving activities.

The Problem Solving Block:

- ☑ focuses on developing strategies to solve a variety of problems
- ☑ includes solving problems in all the content strands: number, operations, and algebraic relationships; geometry; measurement; and data analysis and probability
- ☑ begins with the teacher telling story problems that have very familiar contexts. (As students' reading skills develop, teachers offer problems in written form. As students' writing skills develop, students write their own story problems as well as write explanations of their solution strategies.)
- ☑ requires students to comprehend the problem's information and action and then to represent the action (acting it out, drawing pictures, using objects, using an empty number line, etc.)
- ☑ expects students to find one or more ways to solve the problem
- ☑ provides time for students to talk about their solutions and determine how and why they feel confident about the accuracy of their solutions
- ☑ composes from 20 to 25 minutes of a math hour
- ☑ occurs in heterogeneous or homogeneous large or small group settings





Content and Process Standards

BIG IDEAS: Content

The **Math Content** *big ideas for problem solving block activities include:*

Number, Operations and Algebraic relationships

- Solve problems with story contexts
- Develop counting and computation strategies
- Extend, create and identify patterns

Geometry

- Identify and draw 2 and 3 dimensional shapes
- Talk about the attributes of 2 and 3 dimensional shapes and sort by those attributes, including symmetry
- Solve problems by putting together and taking apart shapes, including flipping, sliding, and rotating shapes as well as creating nets (flat patterns) of rectangular prisms
- Extend, create and identify patterns

Measurement

- Compare and order objects according to attributes
- Learn how to use units to measure attributes
- Talk about the reasons for using the same size of unit when measuring
- Talk about the relationship between the size of the unit and the number of units needed to measure

Data analysis and probability

- Ask a question to be answered by collecting information (data)
- Collect and organize the information (data)
- Create a graph to show the organized information
- Use the graphs to try to answer the original question

For specific grade level information on math content, see the MMSD K-5 Grade Level Mathematics Standards.



Content and Process Standards
BIG IDEAS: Process

*The **Math Process** big ideas for problem solving block activities include:*

Problem solving

- Engage in finding ways to solve a problem
- Create models to show understanding of the problem and to assist in solving it
- Reflect on one's own strategies to continue making sense of the problem solving experience
- Develop the habit of rethinking the solution to assure accuracy

Representation

- Record the thinking steps one takes to solve a problem
- Create graphs to show data
- Learn conventional forms of representing operations and other mathematical ideas

Communication

- Share solution strategies using words, pictures, and numbers
- Create story problems
- Learn mathematical vocabulary

Reasoning and Proof

- Explain the reasoning one uses to draw a conclusion (e.g., these shapes must be the same because when I flip this one over, it matches the other one.)
- Look for patterns (e.g., patterns on a hundred chart or the pattern of even and odd numbers)
- Talk about why things make sense

Connections

- Discuss how two solution strategies are the same and how they are different
- Comment on ways new ideas are alike and different from ideas seen before
- Recognize the value of using different names for numbers to solve a given problem (e.g., solving 56 + 8 can be more efficient if one thinks of 8 as 4 + 4)

For specific grade level information on math processes, see the MMSD K-5 Grade Level Mathematics Standards.

Teaching Activities in the Problem Solving Block

Story Problems: in the Problem Solving Block

There are three considerations when planning:

1. grouping arrangements to meet students' needs
2. selection of the problems
3. adaptations and extensions of the problem solving experiences



Teachers decide on the *grouping arrangements* for problem solving that best meet the needs of their students. Because there are many ways to solve a problem, many problems lend themselves to engaging the entire class at one time. However, when the objective of the lesson is to develop skills in solving a particular problem type, using a particular solution strategy or working with particular number sizes, problem solving activities are more successful when done as focused, small group activities. To use problem solving activities effectively, teachers plan which grouping structure best fits the learning objectives.

When planning *which problems* to pose, teachers use the problem solving activities in the school's curricular resources. Teachers also often write story problems tailored to the experiences and interests of the students, using student names and situations familiar to the class.

Teachers extend or adapt the problem solving experience by writing problems at students' reading levels so students can read and solve problems independently.

The way the page is organized signals to the students whether or not they need to show their thinking steps. There are two ways to format story problem pages.

1. If the problem types or solution strategies are at the students' *instructional* level, the teacher will want to see how the students explain their strategy(ies). On these pages, there should be space available to draw pictures or an empty number line or use arrow language to explain students' thinking steps.
2. Teachers create pages of story problem types and number sizes that are at the student's *independent* level. On these pages, there is no space left for students to write explanations for solution strategies.

   (See examples of page formats in Appendix.)

A Typical Sequence for Story Problem Activities

- ☑ Teacher selects problem types and number sizes to develop a particular group's solution strategies. (See Problem Solving Assessments in Chapter 4)
- ☑ Teacher poses the problem to students.
- ☑ Students model, use counting strategies or mental computations to solve the problem.
- ☑ Students listen as classmates share their problem solving strategies.
- ☑ Teacher uses questions to guide the conversation so students reflect on their own and their classmates' solution strategies.
- ☑ Through discussion, the students reason about and determine the accuracy of their solutions.
- ☑ Students develop their metacognitive skills by reflecting on their strategies and their knowledge of number relationships and math concepts.
- ☑ Students learn ways to use pictures, words, and symbols to record their solution strategies.
- ☑ Students can create and share their own story problems and ask their classmates to solve them and explain their strategies.



When planning the next level of challenge for each student, teachers use the *MMSD K-5 Grade Level Mathematics Standards* to guide the selection of problem types and number sizes. Teachers know that every student will take a unique path in becoming proficient with the problem types, solution strategies and number sizes. An overview of the K-2 development as described in the MMSD standards appears in the following table:

Kindergarten	**First Grade**	**Second Grade**
Problem types: Join, Result Unknown Separate, Result Unknown Multiplication Measurement Division Partitive Division, sharing by 2 *Solution strategies:* Modeling strategies Acting it out Using objects Drawing pictures Counting strategies, Moving into counting on 1 and 2 *Number Sizes:* Modeling strategies: 0 – 20, focus on 0 – 10	*Problem types:* Join and Separate, Result Unknown Multiplication Measurement Division Partitive Division, sharing by 2, 3 and 4 Join and Separate, Change Unknown Compare, Difference Unknown Part, Part, Whole, Whole Unknown *Solution strategies:* Counting strategies Counting on or back 1, 2, 3 Counting by groups of 2, 5, 10 Modeling strategies Acting it out Using objects Drawing pictures Using empty number lines *Number sizes:* Mental computations: Addition - 0 – 10, moving into 0 – 20 Modeling strategies: 0 – 100	*Problem type:* Join and Separate, Result Unknown Multiplication Measurement Division Partitive Division, sharing by 2, 3, 4, 6 and 8 Join and Separate, Change Unknown Compare, Difference Unknown Part, Part, Whole, Whole Unknown Part, Part, Whole, Part Unknown Join and Separate, Start Unknown *Solution strategies:* Counting strategies Counting on or back 1, 2, 3 Counting by groups of 2, 5, 10 Decomposing numbers Using landmarks (10) Using place value concepts Modeling strategies Acting it out Drawing pictures Using objects Using empty number lines Using arrow language *Number sizes:* Mental computations: Addition – Sums from 0 – 20 Place value concepts – 0 – 100 Subtraction – Differences of 1, 2 or 3 Modeling and counting strategies: Numbers beyond 100

One first grade teacher plans for problem-solving instruction in the following way:

Planning Notes – February 12 – *Analyzing this week's work, I see 3 groups for next week's problem-solving instruction: (names are pseudonyms)*

Group 1
Hank, Eliza, Gabrielle, Jonah (Stephen might be able to work with this group.)
They are working with multi-digit numbers using arrow language. They are counting on to solve JRU, SRU, M problems.

Independent work *– multi-digit numbers, JRU, SRU, M problems*

Instructional focus *– JCU, CDU, and MD and PD problems*
Modeling and then representing modeling. It will be important to support their discussion about how the model shows their use of number relationships. They can use arrow language to represent each solution, but the answers appear in different parts of the arrow language representation. Need to emphasize checking again to see if answer makes sense

Group 2
Dave, Cicelia, Jon, Tree, Mari, Pa Lia, Zach (Stephen might be working with this group.)
They are working with numbers to 20. They use cubes to solve and need to be supported in their efforts to show pictures to represent their models.

Independent work *– JRU, SRU, M using numbers to 10*

Instructional focus *– RU, SRU, Their number work should focus on strengthening counting on strategies for sums to 20 (bead string, one push and then count on 1, 2, 3, covered tasks, etc, numeral tracks). M, and MD. Work with multiples of 2 and 5. Model and support thinking of counting the model more efficiently, using their count by 2's and 5's knowledge*

Group 3
Quinn, Sabrina, Renie
They are counting on to solve problems and are using the empty number line to represent their counting on strategies.

Independent work *– JRU, SRU, M (how are they representing M?) numbers to 20.*

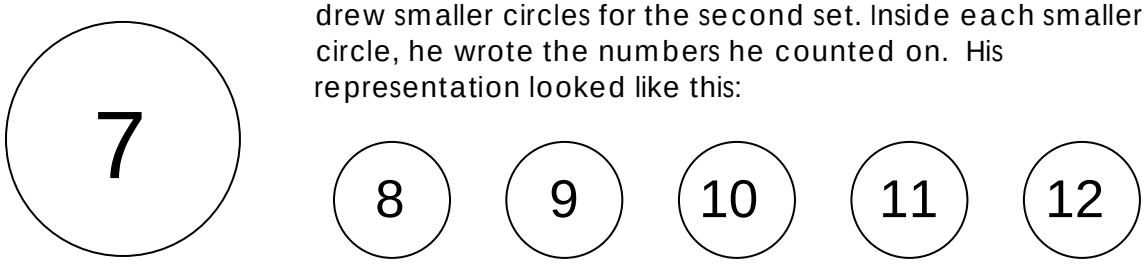
Instructional focus *– JRU, SRU, M, MD using multi-digit numbers, modeling with tens and ones. JCU, CDU, SCU using numbers to 20, working to decompose one addend and learning to use the empty number line to represent their counting on strategies for these problem types.*

Teaching students ways to write down their "thinking steps"

Using Drawings

Students learn to represent the thinking steps they use to solve a problem in many ways. When students model solutions, they represent their work by drawing pictures of the objects they used. Usually students are satisfied with just drawing circles or squares to indicate the objects they have counted. When students insist on taking time to draw the exact object they used, a teacher might point out that they are making a drawing for math. All they need to do for a math drawing is to draw shapes to count.

As students begin to use counting strategies, it is a challenge for them to figure out ways to show their counting strategies. Having used a counting on strategy, one student showed his thinking in this way. He wrote a number in a circle. Then he drew smaller circles for the second set. Inside each smaller circle, he wrote the numbers he counted on. His representation looked like this:



Another first grader wrote a number and then drew a hand with fingers, labeling each finger as she counted on.

Using Empty Number Lines

When students are comfortable using counting on strategies, they are ready to learn how to use an empty number line, a slightly more abstract way of representing their counting strategies. The power of the *empty number line* is that as a student constructs and uses it, the student reinforces his/her understanding of number relationships.

Using Arrow Language Notation

When students can use place value concepts to work with multi-digit numbers, they can begin using *arrow language notation*, an even more abstract way of representing their thinking steps.

Using a Sequence of Number Sentences

When students work with multi-digit numbers, they use a sequence of number sentences (equations) to describe their series of mental computations. For instance, one student wrote this sequence for a separate, result unknown problem:

72 – 2 = 70
70 – 30 = 40
40 – 5 = 35

Using the Empty Number Line to Represent Counting Strategies

Once a student consistently uses counting strategies, the teacher will want to show a student how to use an empty number line to represent her/his thinking.

Counting On By Ones

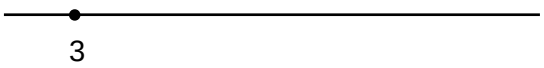
Given this problem: *"Harvey had 3 stickers. His friend gave him 2 more. How many stickers did Harvey have then?"* A student will start at 3 and count 2 more, ending at 5.

After the student has shown or explained the way he/she counted on, the teacher takes the opportunity to demonstrate how to create an empty number and use it to show the student's thinking steps.

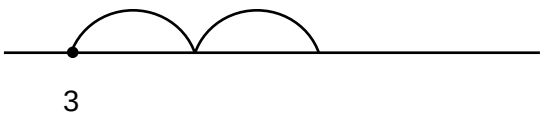
The teacher may say, *"I have a way to show just how you did that."* The teacher will then explain and demonstrate:

Draw a horizontal line.

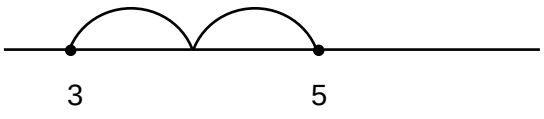
Add a dot on the line and write a 3 below the dot.



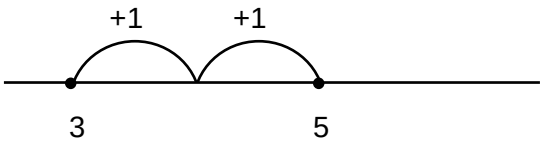
Then draw 2 arches to the right (because the child is counting on).



Add a dot at the stopping point and write a 5 below that dot.



The last thing to do is to label the size of the arches (jumps).



The teacher might recap by saying, *"You started at 3 and then counted on one more and then one more. You ended at 5. Remember to write how big your jumps are."*

Counting Back By Ones

Given this problem: "*Tenzin had 28 stickers. She gave away 3 stickers. How many stickers did Tenzin have then?*", a student who has the number sense to be able to count back will start at 28 and count back 3.

After the student has shown or explained the way he/she counted back, the teacher takes the opportunity to demonstrate how to create an empty number line and use it to show the student's thinking steps for this problem.

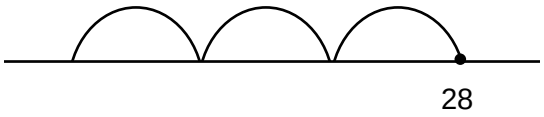
The teacher may say, *"I have a way to show just how you did that."* The teacher would then explain and demonstrate:

Draw a horizontal line.

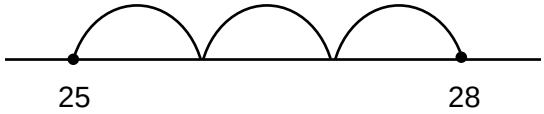
Add a dot on the line and write a 28 below the dot.



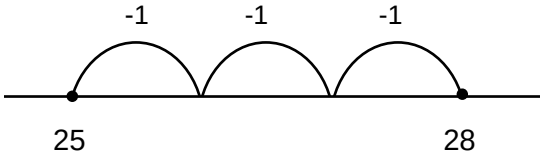
Then draw 3 arches to the left (because the child is counting back).



Add a dot at the stopping point and write a 25 below that dot.



The last thing to do is to label the size of the arches (jumps).

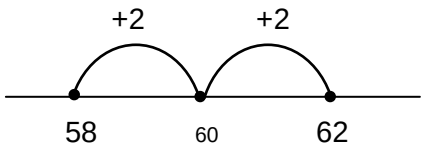


The teacher might recap by saying, *"You started at 28 and then counted back one, then one more and then one more. You ended at 25. Remember to write how big your jumps are."*

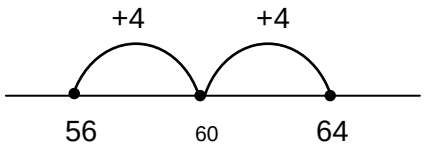
Counting On Or Back By Number Chunks:

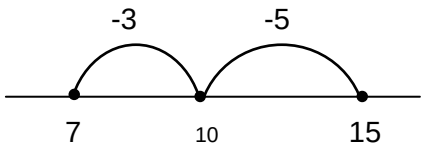
Students begin to think more efficiently and count on or back in quantities other than one. They might count on or back by 2's. They will use 10 or another decade number as benchmarks to work to or through. When a student has developed this understanding of number relationships, he/she will seldom use manipulatives but rather use the empty number line as the model to track thinking steps.

For instance, given a problem that calls for adding 4 more, a student might add 2 and then 2 more. The empty number line representation would look like this:



Students who recognize the power in working to, through or from a decade number will show their thinking using an empty number line. Here are some examples of how these students might represent their thinking:

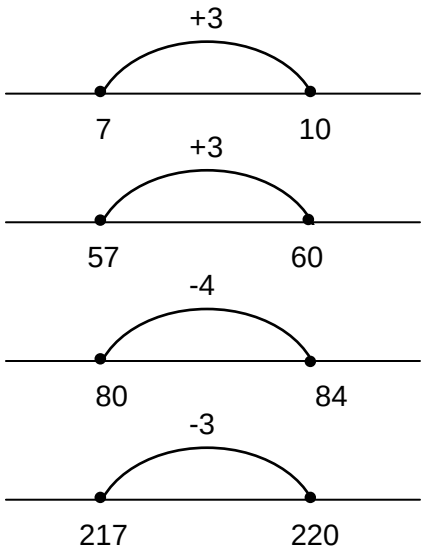




Moving to the Decade

Students learn to use 10 and 5 as landmarks. As they see numbers in relation to these landmarks, they recognize them as part, part, whole relationships. For instance, 6 is 4 away from 10, therefore 6 and 4 make 10. This understanding of number relationships is used to understand the relationships of non-decade numbers to the next or the previous decade. The empty number line can be used to represent these relationships.

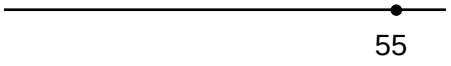
Here is a series of empty number lines that represent this thinking:



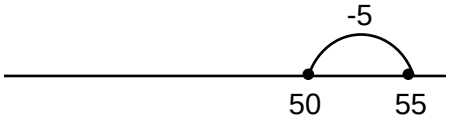
Students use their understanding of how the numbers relate to landmarks, sometimes called *friendly numbers*, to help them calculate across a decade.

Given the problem: *"Enanu made 55 peanut butter cookies and her brother ate 7. How many cookies did she have then?",* a child first subtracts 5 to reach the decade number and then knows that 2 less than 50 is 48. Here is how she would show that:

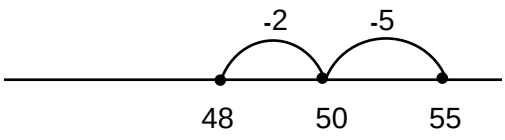
Draw a horizontal line, add a dot and label it 55.



Indicate a jump to the decade.



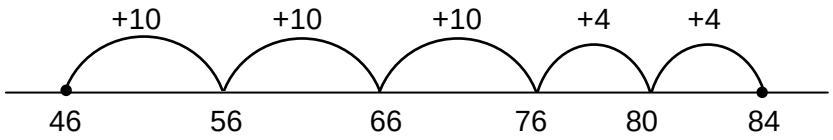
Indicate the next jump and label the dot.



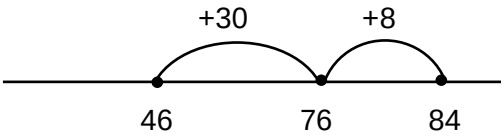
Working With Tens and Ones

As students' understanding of place value develops, they learn to compute using place values of numbers. They can use an empty number line to record their thinking steps. The empty number line representation reflects how the student worked with the place values.

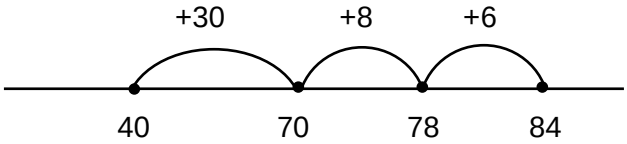
For this problem: "*Sal got 38 points on his first try. On his second try, he got 46 points. How many points did Sal get in all?"*, a student might begin with 46, then count on by tens 56, 66, 76, and then add 8 more, 80, 84. This is how the student would use the number line to represent that strategy:



Another student might think 46 and 30 is 76 and then add 8. This is an empty number line representation of these thinking steps.



Another student might think 40 and 30 is 70 and then add 8 and then 6. These thinking steps look like this on an empty number line:



The empty number line is a tool that reinforces visualizing the spatial organization of numbers on a horizontal number line. It can be adapted to meet students' needs as their understandings of place value concepts develop from using objects for modeling to using mental computation strategies. When students first learn to use the empty number line, they are showing the steps that they used to count the tens and ones models they constructed. As students' fluency with place value concepts develops, they use the empty number line to track each step so they do not have to store so much information in their short-term memory. Eventually, students decompose multi-digit numbers into tens and ones, compute each and then add the partial sums. They use a sequence of equations to show their thinking steps.

Using Arrow Language to Represent Thinking Steps

When students develop place value concepts to compute with multi-digit numbers, they use models or mental constructs to:

- count on by tens from a non-decade number
- count on by sets of decades
- decompose multi-digit numbers into tens and ones and compute each and then add the partial sums

To track their thinking steps as they work with place values or to capture their thinking processes on paper, students can use arrow language notation.

After listening to a student share how she/he worked with place values to compute a solution, the teacher shows the student how to represent her/his place value thinking.

Counting on or back by tens

Given this problem: *"Tenzin made 37 peanut butter cookies and 48 chocolate chip cookies. How many cookies did Tenzin make altogether?",* a student may put out place value blocks and count or use mental constructs to think: 48, ten more 58, ten more 68, ten more 78, two more 80, and 5 more 85. Tenzin made 85 cookies in all.

After listening to this explanation, the teacher may say, "I have a way to show those thinking steps." Then the teacher will explain:

"You started with 48. Here is the 48," and she writes

48

"Then you added 10 more. Here is a way to show that step." and she writes

$$48 \xrightarrow{+10} 58$$

"Then you added 10 more and 10 more."

$$48 \xrightarrow{+10} 58 \xrightarrow{+10} 68 \xrightarrow{+10} 78$$

"And then you added ten to make 80 and then 5 to make 85."

$$48 \xrightarrow{+10} 58 \xrightarrow{+10} 68 \xrightarrow{+10} 78 \xrightarrow{+2} 80 \xrightarrow{+5} 85$$

Adding a set of decades to a non-decade number

If, given the same problem: *"Tenzin made 37 peanut butter cookies and 48 chocolate chip cookies. How many cookies did Tenzin make altogether?"*, the student shares that he started with 48 and added 30 and then 7, the teacher may say "I have a way to show that thinking." Then the teacher will explain:

> "You started with 48. Here is the 48," and she writes
>
> 48.
>
> "Then you added 30 more. Here is a way to show that step," and she writes

$$48 \xrightarrow{+30} 78 \xrightarrow{+7} 85$$

Adding the tens and then the ones

If, given the same problem as above, the student shares that she thought about the tens and added 40 and 30 to make 70; then thought about the ones and added 8 and then 7, the teacher may say "I have a way to show that thinking."
Then the teacher will explain:

> "You started with 40 and added 30 and 8 and then 7."

$$40 \xrightarrow{+30} 70 \xrightarrow{+8} 78 \xrightarrow{+7} 85$$

Eventually, students decompose multi-digit numbers into tens and ones and compute each and then add the partial sums. They use a sequence of equations to show their thinking steps.

Geometry Problems in the Problem Solving Block

These problems challenge students to think about spatial relationships, see shapes within shapes, sort by attributes (including symmetry), and examine the results of flipping, sliding and turning shapes. Typical geometry problems in the primary grades include filling space using pattern blocks, tangrams, and 1 inch square tiles. Students also learn to extend, create and identify patterns.

Teachers use the school's curricular materials as a source for geometry problems. Also, in the appendix, there is a sequence of K – 2 problem solving geometry activities using square tiles for kindergarten, first and second grade. Additional geometry problems are listed in *For more information* at the end of this chapter.

Measurement problems in the Problem Solving Block

Manipulating units to measure length, weight, capacity and time and learning how to use tools to measure these attributes are the major focus of lessons about measurement. Measurements can be used as the context for story problems. For instance, a child could look at a table of the measurements of her/his bean plant and solve a change unknown problem to find out how much it grew in a week. The school's curricular materials for both math and science are sources for measurement problems. Additional measurement problems are listed in *For more information* at the end of this chapter.



For more information:

Number, Operations and Algebraic relationships

Andrews, A. G. & Trafton, P. (2002). *Little kids – powerful problem solvers: Math stories from a kindergarten classroom.* Portsmouth, NH: Heinemann.

Dacey, L.S., & Eston, R. (1999). *Growing mathematical ideas in kindergarten.* Sausalito, CA: Math Solutions Publications.

Carpenter, T. P., Fennema, E., Loef Franke, M., Levi, L., & Empson, S. B. (1999). *Children's mathematics: Cognitively guided instruction*. Portsmouth, NH: Heinemann.

Greenes, C., Cavanagh, M., Dacey, L., Findell, C., & Small, M. (2001). *Navigating through algebra in prekindergarten through grade 2*. Reston, VA: National Council of Teachers of Mathematics.

Greenes, C. E., Dacey, L., Cavanagh, M., Findell, C. R., Sheffield, L. J., & Small, M. (2004). *Navigating through problem solving and reasoning in grade 1.* Reston, VA: National Council of Teachers of Mathematics.

Greenes, C. E., Dacey, L., Cavanagh, M., Findell, C. R., Sheffield, L. J., & Small, M. (2003). *Navigating through problem solving and reasoning in kindergarten.* Reston, VA: National Council of Teachers of Mathematics.

Madison Metropolitan School District. (2004). *MMSD mathematics Grade Level K-5 standards*. Madison,WI: Retrieved May 1, 2009 from http://mathweb.madison.k12.wi.us/node/268

Pilar, R. (2004). Spanish-English language issues in the mathematics classroom. In Ortiz-Franco, L., Hernandez, N. G., & De La Crus, Y. (Eds.). *Changing the faces of mathematics perspectives on Latinos*. (pp. 23-33). Reston, VA: National Council of Teachers of Mathematics.

Sheffield, Stephanie. (2001). *Lessons for first grade*. Sausalito, CA: Math Solutions Publications.

Small, M., Sheffield, L. J., Cavanagh, M., Dacey, L., Findell, C. R. & Greenes, C. E. (2004). *Navigating through problem solving and reasoning in grade 2.* Reston, VA: National Council of Teachers of Mathematics.

von Rotz, L., & Burns, M. (2002). *Lessons for algebraic thinking*. Sausalito, CA: Math Solutions Publications.

Geometry

Bachman, V. (2003). *First-grade math: A month-to-month guide.* Sausalito, CA: Math Solutions Publications.

Confer, C. (2002). *Math by all means: Geometry grades 1-2.* Sausalito, CA: Math Solutions Publications.

Dana, M. *Geometry: Experimenting with shapes and spatial relationships-grade 1*. Grand Rapids, MI: Instructional Fair, Inc. (Out of print, see an elementary math resource teacher)

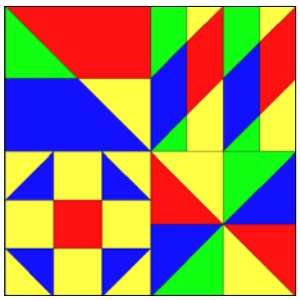
Dana, M. *Geometry: Experimenting with shapes and spatial relationships-grade 2*. Grand Rapids, MI: Instructional Fair, Inc. (Out of print, see an elementary math resource teacher)

Findell, C. R. Small, M., Cavanagh, M., Dacey, L., Greenes, C. R. & Sheffied, L. J. (2001). *Navigating through geometry in prekindergarten – grade 2.* Reston, VA: National Council of Teachers of Mathematics.

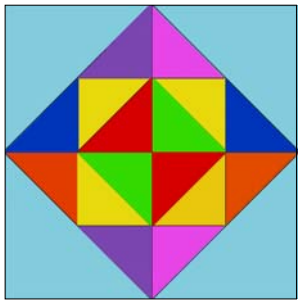
Litton, N. (2003). *Second-grade math: A month-to-month guide.* Sausalito, CA: Math Solutions Publications.

Measurement

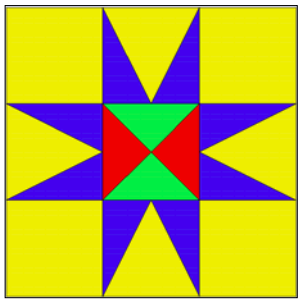
Dacey, L., Cavanagh, M., Findell, C. R., Greenes, C. E., Sheffied, L. J. & Small, M. (2003). *Navigating through measurement In prekindergarten – grade 2.* Reston, VA: National Council of Teachers of Mathematics.



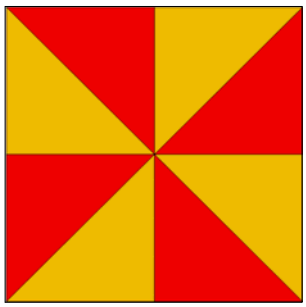
PROBLEM SOLVING



NUMBER WORK



INSPECTING EQUATIONS

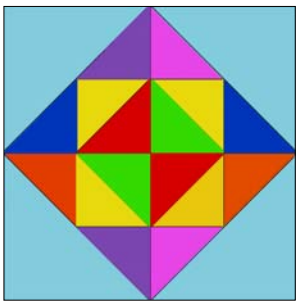


FLUENCY & MAINTENANCE

Chapter 7

Number Work

Learning activities in each block
support the learning activities in the other blocks.



Number Work
15-25 minutes

Number Work Block

Number work activities challenge students to focus on numbers and solve problems without a story context. The focus is on developing students' sense of number. Number work can include such activities as: ways to make a number; what do you know about a given number; function machines; numeric patterns; and patterns on a hundred chart. Number work is most productive when it becomes routine. Because number work often involves activities with multiple entry points and a search for a diverse set of responses, it frequently can be used as a whole class activity.

The Number Work Block:

- ☑ focuses on developing skills in thinking about number concepts
- ☑ can often be posed as a problem to solve
- ☑ uses activities that often have multiple entry points or more than one answer
- ☑ composes from 15 - 25 minutes of a math hour
- ☑ occurs as a heterogeneous large group or an individual activity





Content and Process Standards

BIG IDEAS: Content

The **Math Content** *big ideas for number work block include:*

Number, Operations and Algebraic Reasoning

- Learn that numbers have many names
- Develop a flexible use of counting sequences
- Develop an understanding of number relationships such as:

  Larger (greater) than, smaller (less) than
  right before ___, right after ___
  2 before, 2 after
  how far from 0, 5, 10, 15, 20 etc.

- Learn place value concepts
- Develop counting and computation strategies
- Recognize and generalize patterns when counting and computing

Geometry

- Determine how many of one shape can make another
- Divide geometric shapes into fractional parts

Measurement

- Count the number of units used to measure
- Investigate the relationship between units (e.g.,12 inches is the same as 1 foot)

Data analysis and probability

- Sort, count and graph sets of objects
- Compare quantities represented on a graph

For specific grade level information on math content, see the MMSD K-5 Grade Level Mathematics Standards.



Content and Process Standards

BIG IDEAS: Process

The **Math Process** *big ideas for number work block activities include:*

Problem solving

- Engage in finding many ways to solve a problem
- Reflect on one's own understanding of number relationships

Representation

- Record the thinking steps one takes to solve a problem
- Create graphs to show data
- Learn conventional forms of representing operations and other mathematical ideas

Communication

- Share solution strategies using pictures and numbers
- Learn mathematical vocabulary

Reasoning and Proof

- Explain the reasoning one uses to draw a conclusion (e.g., I know that 5 + 5 is 10. 6 is 1 more than 5, so 5 + 6 must be one more than 10.)
- Look for patterns (e.g., patterns on a hundred chart or the pattern of even and odd numbers)
- Talk about why things make sense

Connections

- Discuss how two number representations are alike and different
- Comment on ways new ideas are alike and different from ideas seen before

For specific grade level information on math processes, see the MMSD K-5 Grade Level Mathematics Standards.

Teaching Activities in the Number Work Block

Number work activities are most effective when they become daily routines. If students are familiar with the format of the activities, they can focus their attention on learning about the number relationships and concepts involved in the activities rather than on how to do the tasks.

Teachers select the number work activities that best meet the needs of their students. When beginning to teach a particular number work activity, the teacher guides the class through the activity every day. As the students participate with greater independence, the teacher begins to ask more probing questions in order to change the focus from learning the activity itself to learning about the number relationships and mathematical concepts.

There are a wide variety of activities that provide excellent number work experiences. Teachers should plan for a balance of activities. As students become totally independent with a particular activity, the teacher moves that activity into the fluency & maintenance part of the math hour and introduces a new number work activity. Teachers find ideas for number work activities in the school's curricular resources.

Number Work activities tend to be divergent in nature and allow for multiple responses, lending themselves to whole class activities. However, all of the activities can be used with a large group, small group, or as individual practice activities depending upon the choice of number size and the students' levels of proficiency.



The activities described in this chapter are listed in two ways:

1. *Level of Complexity List*

   The activities are listed from those most appropriate for kindergarten students to those most appropriate for second grade students. All of the activities in the list can be used at more than one grade level by adjusting the size or range of numbers used.

2. *Alphabetical Order List*

   The activities are listed in alphabetical order with brief teaching suggestions.

Level of Complexity List
(See the alphabetical order list on the following pages for descriptions of the activities and brief teaching suggestions)

Counting Chants and Poems
Writing Numerals
Counting with the 20 Bead String
Count Some More
Counting By ____ Toss
After & Before
What Do You Know About _____?
Close To _____
Longer Than, Shorter Than
Ways To Make ____
Coins for the Day!
Reading Counting Books
Hundred Chart Patterns
Compare Empty Number Line Representations
Rainbow Numbers
Fact Strategies
Reading and Interpreting Graphs
Card Games
Looking for Tens
What's the Rule?
Compare Arrow Language Representations
Numeric Patterns
Estimating
Which Number Does Not Belong?
Guess the Sorting Labels

Alphabetical Order List

The following activities are listed in alphabetical order. Refer to the *Level of Complexity* list on the previous page to identify more or less complex activities. Each activity can be adapted to a different grade level by changing the sizes of numbers used.



After & Before

Topic: forward and backward counting word sequence, fluency

What to do:

- ☑ The group sits in circle.
- ☑ One student picks a card (or two cards if working on 2 digit numbers, etc), shows them to student on the left, and says 1 after (or 2 after or 1 before, whatever you choose, or could be student choice).
- ☑ Student tells the number that is 1 after (etc.) and picks a card and repeats the play.
- ☑ Continue until all students have had a turn.



Card Games

Topic: number relationships and fact strategies

What to do:

- ☑ Teach the rules for the games that will be used in the Fluency & Maintenance Block.
- ☑ See Chapter 9, Fluency & Maintenance Block for game rules.



Close To ____
Topic: number relationships

What to do:

- ☑ Each student has a card.
- ☑ Class decides upon a 2 digit target number.
- ☑ Pairs of students put their cards together to make a two-digit number that comes closest to the target number.
- ☑ When students are fluent in making two digit numbers, make the target number a three digit number.



Coins for the Day!
Topic: knowing and adding values of coins

What to do:

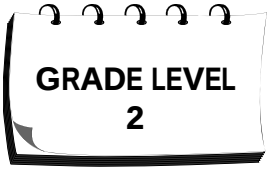
- ☑ Students work together, or one student each day, to put together a selection of coins that will match the number of the total days in school.
- ☑ Variation: Find the selection that uses the least number of coins, the most number of nickels, etc.



Compare Arrow Language Representations
Topic: representing the operations of addition or subtraction

What to do:

- ☑ Write two arrow language representations large enough for the entire group to see.
- ☑ Ask students to tell the ways the two arrow language representations are the same and list their responses.
- ☑ Ask students to tell the ways the two arrow language representations are different and list their responses.



Compare Empty Number Line Representations
Topic: representing the operations of addition or subtraction

What to do:

- ☑ Write two empty number line representations large enough for the entire group to see.
- ☑ Ask students to tell the ways the two empty number line representations are the same and list their responses.
- ☑ Ask students to tell the ways the two empty number line representations are different and list their responses.



Counting Chants and Poems
Topic: forward and backward counting word sequences

What to do:

- ☑ Post the poem on chart paper.
- ☑ Read and memorize the poem.
- ☑ Highlight the counting word sequence.

(See traditional counting rhymes resource in Appendix)



Counting By ____ Toss
Topic: fluency in counting on)

What to do:

- ☑ Stand in a circle.
- ☑ Roll a prepared number cube to determine if adding on 1, 2, or 3.
- ☑ First student chooses a single digit number as the starting number.
- ☑ Student tosses the beanbag and next student catches the beanbag and states the new number.
- ☑ Play continues until all students have had a turn.
- ☑ Variation: First student chooses a 2-digit number.



Count Some More
Topic: fluency in counting on and back

What to do:

- ☑ One student picks a card from a container of cards.
- ☑ The student identifies the number on the card.
- ☑ The class continues counting on (or back) from that number.

Counting with the 20 Bead String



Topic: counting sequences and conceptual subitizing

What to do: See 20 Bead String page in Appendix.



Estimating
Topic: number relationships and visualizing quantities

What to do:

- ☑ Find two jars that are transparent and exactly the same.
- ☑ Have a collection of items in one jar and an appropriate small quantity (2, 3, 5, 10) in another jar to use as a reference. Tell the students how many object are in this jar.
- ☑ Ask students to think about how much space is used by the smaller quantity of objects. They should use that understanding to help them think about how many there might be in the larger jar.
- ☑ After students share their thinking, count the quantity in the larger jar.
- ☑ Discuss what estimates are close. Talk about the reasons. Ask students to share and reflect on their strategies.



Fact Strategies
Topic: making a ten, using number relationships

What to do:

- ☑ Show the class a set of open number sentences (equations).

$10 = 7 + \square$

$15 + \square = 20$

$\square + 38 = 40$

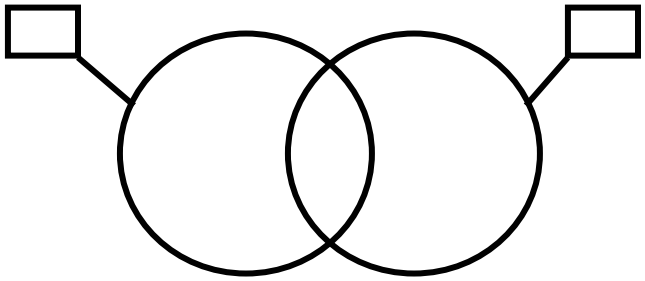
- ☑ Ask students "What strategies would you use to determine the numbers that make these number sentences true?" Students might say: counting on, using landmarks (friendly numbers) or other things they know about the numbers that help them determine the answer without counting.

GRADE LEVEL
2

Guess the Sorting Labels
Topic: number relationships and odd-even classification

What to do:

- ☑ Put a Venn diagram on the board with hidden labels for each circle.



- ☑ Rules can be things such as odd numbers or even numbers, greater than 5, greater than 10, less than 20, has 2 digits, etc.
- ☑ Students take turns guessing a number.
- ☑ If the student has guessed correctly, the teacher puts the number in the correct place on the Venn diagram. (Remember the universal set outside of the circles!)
- ☑ As the set of numbers grows in each area of the Venn Diagram, students may first make a number guess and then conjecture about the label for the circle after guessing a new number.
- ☑ Continue playing until the labels for each circle can be determined.
- ☑ Check to make sure that each number in the diagram meets the sorting criterion.

Sorting labels can include:

Odd numbers, Even numbers

Before 5, After 5

1 digit numbers, 2 digit numbers

Even numbers, Decade numbers

2 digit numbers, 7 in the ones place

Even numbers in the ones place, Multiples of 3

Even numbers, Decade numbers



Hundred Chart Patterns
Topic: patterns and number relationships

What to do:

- ☑ Ask students to look for patterns on the 100 chart.
- ☑ Keep a chart describing the patterns.
- ☑ Possibilities include:
  - even numbers,
  - odd numbers,
  - numbers ending in 5,
  - numbers ending in 0,
  - counting by 3's, etc.



Longer Than, Shorter Than (Heavier Than, Holds More Than)
Topic: comparison of attributes

What to do:

- ☑ Ask students to think about (find) things in the room that are longer than (or shorter than) a given number of non-standard units (cubes, measuring worms, paper clips, etc.).
- ☑ Make a chart of the things they identify. If possible, check by placing units next to the objects in order to compare lengths.



Looking for Tens
Topic: part, part, whole relationships to make 10

Can be used in first or second grade depending upon the range of numbers from which to choose.

What to do:

- ☑ Roll a cube with numbers three times or choose 3 cards.
- ☑ Write those three numbers large enough for the entire group to see.
- ☑ Ask the group to share their ideas on how to put the numbers together to be able to add them most easily.
- ☑ Repeat several times.
- ☑ Variation: When fluent with 3 addends, go to 4 addends, and then to 5 addends

GRADE LEVEL
1-2

Numeric Patterns
Topic: number relationships and repeating and growing patterns

What to do:

- ☑ Write a numeric pattern large enough for the entire group to see it. For example: 1, 5, 9, 13, ...
- ☑ Ask students to think about how the numbers change from one number to the next. (In the 1, 5, 9, 13 . . . sequence, each number is 4 more than the previous number.)
- ☑ As students share their ideas, check them out to see if they work consistently from one number to the next.
- ☑ Students identify the rule and keep the pattern going. (In the 1, 5, 9, 13 sequence, the rule is + 4.)
- ☑ Extension: Once the change pattern is identified, ask if a particular number will be in the pattern if you keep the pattern going.

GRADE LEVEL
1-2

Rainbow Numbers
Topic: part, part, whole relationships to make 10

What to do:

- ☑ Create a number line from 0 to ___. (Let's say 7.)
- ☑ Ask students what they would add to ____ (0) to make a total of ____ (7).
- ☑ Connect the 7 and the 0 on the number line with a tall arch.
- ☑ Ask the students what they would add to the next smaller number (6) to make a total of ___ (7).
- ☑ Connect that pair of numbers with an arch.
- ☑ Continue until all numbers are connected. Using a different color for each arch makes this more interesting.
- ☑ For more information, see Rainbow Numbers page in Appendix.

GRADE LEVEL
K-1-2

Reading Counting Books
Topic: forward and backward counting sequences and number order

What to do:

- ☑ Have a variety of counting books available.
- ☑ A student selects a book,
- ☑ The student reads or looks at the book to learn about the counting sequence in the book and the things being counted.
- ☑ Extension: The student can complete a response page. See reader response page examples in Appendix.

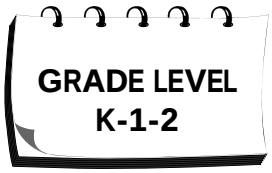


GRADE LEVEL
K-1-2

Reading and Interpreting Graphs
Topic: counting, representing and comparing quantities

What to do:

- ☑ Use the endless opportunities you have to create graphs such as: graphing hot lunch counts, colors of shoes, favorite authors of picture books, etc.
- ☑ Pose questions such as— Which has the most? Least? Which is more (greater)? How much more?



Ways To Make _____ (Other Names For _____)
Topic: number relationships and relationships between operations

What to do:

- ☑ Students offer ways to add, subtract, multiply, or divide to make a target quantity.
- ☑ At first, the class will create a random list that is good for a discussion comparing/contrasting the expressions.
- ☑ The next step is using patterns, i.e. 9 + 1, 8 + 2, 7 + 3, 11-1, 12-2, etc.).
- ☑ Eventually, students can generate a list independently and the teacher can foster a particular kind of response by asking for that kind to be shared. (E.g., Who has thought of one that uses a fraction? Who has thought of one that uses more than one operation?)
- ☑ Using a 10 ten frames chart to keep track of the number of days in school offers students a powerful place value representation of multi digit numbers. Students use the 10 tens frame chart to work with 10's and 1's, 5's or combinations to 10 when finding ways to make a target number.



What's the Rule? (Function Machine)
Topic: number relationships and patterns

What to do:

- ☑ Input any single or two-digit number, output 1 more (or 2 or 3 more, or 1 or 2 or 3 less).
- ☑ Have students offer numbers to input.
- ☑ Students observe the relationships and decide upon the rule the machine is following.
- ☑ When students are fluent with these relationships, add 10 more and 10 less rules.
- ☑ When students are fluent with all of the above patterns, move to 5 more and 5 less.

GRADE LEVEL
1-2

What Do You Know About ____?
Topic: number relationships

What to do:

- ☑ Teacher asks the students to share what they know about a given number and writes the responses on a chart.
- ☑ Elicit responses from all students.
- ☑ Responses can be a personal reference, such as my brother is 10.
- ☑ Encourage statements that indicate a relationship, such as 10 is 2 more than 8.
- ☑ When the group has considered the same number a month or so later, bring out the chart created earlier and compare responses.

GRADE LEVEL
1-2

Which Number Does Not Belong?
Topic: number relationships

What to do:

- ☑ Write a set of numbers on the board like this one:

6	15
10	12

- ☑ Ask students "Which number does not belong with the others? Why?"
- ☑ These kinds of problems are very complex. They are fun to do every now and then. Working on them builds perseverance. These problems challenge students to think from more than one perspective.

GRADE LEVEL
K

Writing Numerals
Topic: identification and numeral formation

What to do:

- ☑ Show students how to form the numerals
- ☑ Set up centers for practice that include materials ***(check for student allergies)*** such as:



- Shaving cream on trays or directly on tables
- Hand cream on individual trays
- Finger paint
- Chalk and individual chalkboards
- Draw with your finger on a partner's back; the partner "feels" the numeral and then writes it on a chalkboard to show you
- Dry erase markers, on individual white boards
- Dry erase markers on classroom windows (check with custodian)
- Dry erase markers or dry erase crayons on plastic pockets with number models inside (use leftover laminate to create a plastic pocket over a 5" x 12" piece of colored tagboard
- Baking soda, half inch layer on individual trays
- Damp sand in sensory table
- Rainbow crayons
- Use regular crayons to trace and retrace over a single numeral shape, creating rainbow colored numerals
- Crayons and paper on bumpy plastic surface (use bumpy plastic ceiling light cover sheets, cut into 10" x 12" pieces; tape edges for safety)
- Wiggle pens
- Magnadoodles
- Squishy bags (hair gel or Crisco, colored with food coloring and sealed inside heavy duty, gallon size freezer bags or sealed food saver bags)
- Wikki stix
- Lightly oiled cooked noodles
- Play dough, with numeral models drawn on plastic sheets for kids to copy or match
- Magic slates

For more information:

Brizuela, B. M. (2004). *Mathematical development in young children.* New York, NY: Teachers College Press.

Confer, C. (2005). *Teaching number sense: Grade 1*. Sausalito, CA: Math Solutions Publications.

Confer, C. (2005). *Teaching number sense: Kindergarten.* Sausalito, CA: Math Solutions Publications.

Dana, M. *Inside-out math problems: Investigate number relationships & operations.* Grand Rapids, MI: Instructional Fair, Inc.

Hope, J. A., Leutzinger, L, Reys, B. J. & Reys, R. E. (1988). *Mental math in the primary grades*. Parsippany, NJ: Pearson Learning Group.

Kamii, C. (2004). *Young children continue to reinvent arithmetic 2nd edition*. New York, NY: Teachers College Press.

Richardson, K. (1999). *Developing number concepts: Counting, comparing, and pattern.* Parsippany, NJ: Pearson Education, Inc.

Richardson, K. (1999). *Developing number concepts: Addition and subtraction.* Parsippany, NJ: Pearson Education, Inc.

Richardson, K. (1999). *Developing number concepts: Place value, multiplication, and division.* Parsippany, NJ: Pearson Education, Inc.

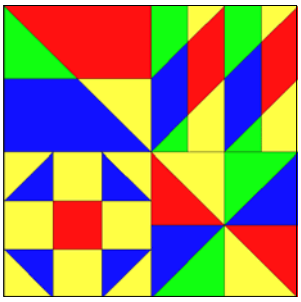
Sharton, S. (2005). *Teaching number sense: Grade 2*. Sausalito, CA: Math Solutions Publications.

Sheffield, Stephanie. (2002). *Lessons for introducing place value*. Sausalito, CA: Math Solutions Publications.

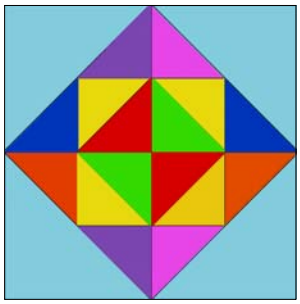
Trafton, P. R., & Thiessen, D. (1999). *Learning through problems: Number sense and computational strategies*. Portsmouth, NH: Heinemann.

Van de Walle, J. A. & Lovin, L. H. (2006). *Teaching student-centered mathematics: Grades K-3.* Boston, MA: Allyn and Bacon Pearson Education, Inc.

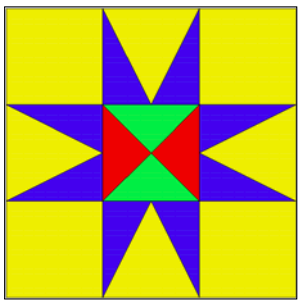
Wheatley, G. H. & Reynolds, A. M. (1999). *Coming to know number: A mathematics activity resource for elementary school teachers.* Tallahassee, FL: Mathematics Learning.



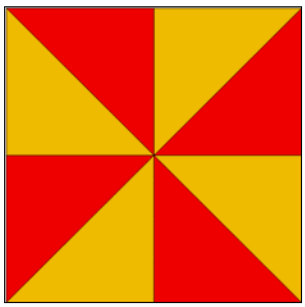
PROBLEM SOLVING



NUMBER WORK



INSPECTING EQUATIONS

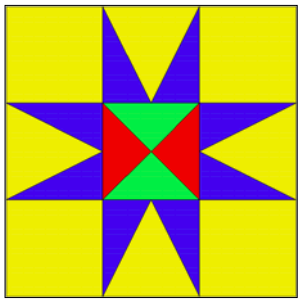


FLUENCY & MAINTENANCE

Chapter 8

Inspecting Equations

Learning activities in each block
support the learning activities in the other blocks.



Inspecting Equations
About 5 minutes

Inspecting Equations Block

Inspecting equations activities focus on learning about equality and how symbols are used to express equality relationships. Students:

- ☑ discuss whether a number sentence expresses a true or false statement of equality
- ☑ use number relationships to reason about equality relationships
- ☑ identify what number(s) replace the unknown quantity to make the number sentence a true statement of equality
- ☑ justify their positions
- ☑ recognize patterns and make conjectures about number properties

Because of the multiple entry points and open-ended nature of inspecting equations activities, they can often be used as whole class activities.

The Inspecting Equations Block

- ☑ focuses on concepts of equality
- ☑ asks students to analyze conventional symbols of quantity, operations, and relationships
- ☑ encourages students to use number relationships as well as computation to confirm equality relationships
- ☑ composes about 5 minutes of a math hour
- ☑ occurs as a heterogeneous large or small group or as an individual activity

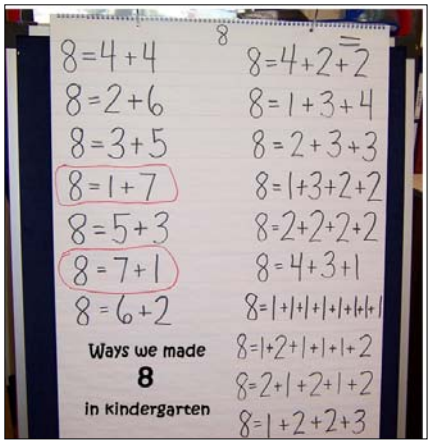
Content and Process Standards

BIG IDEAS: Content

*The **Math Content** big ideas for inspecting equations block activities include:*

Number, Operations and Algebraic Relationships

- Understand the equal sign is a statement of equality
- Understand and use conventional symbolic notation
- Use number relationships to compute more fluently
- Compute accurately



In kindergarten, students are introduced to the equal sign. They learn that it means "the same (amount) as." Teachers encourage students to say "8 is the same (amount) as 7+1" instead of "8 equals 7+1." Another way kindergarten teachers set the stage for future inspecting equations activities is to record students' efforts to find ways to make a given number by writing the sum on the left side of the equal sign and the addends on the right. This makes sense since the students know the quantity they start with and work to determine the ways to make two (or more) addends.

In first grade, students develop their understandings of how to represent statements of equality. They inspect equations such as:

$7 = 7$

$8 + 3 + 5$

$8 + 2 = 2 + 7$

$4 + 2 = \square + 2$

Second graders expand their understanding of equations. They begin to use relationships as well as computation to reason about the truth of equality statements. They inspect equations such as:

$6+4 = 10 + 0$

$6 + 4 = 5 + 5$

$3 + 7 = 7 + 3$

$3 + 4 = 3 + 3 + \square$

For specific grade level information on math content, see the MMSD K-5 Grade Level Mathematics Standards.



Content and Process Standards
BIG IDEAS: Process

*The **Math Process** big ideas for inspecting equations block activities include:*

Problem-solving

- Reflect on one's own strategies to continue making sense of the problem solving experience
- Develop the habit of rethinking the solution to assure accuracy

Representation

- Learn conventional forms of representing operations and number relationships

Communication

- Share solution strategies using words and numbers
- Learn mathematical vocabulary

Reasoning and Proof

- Explain the reasoning one uses to draw a conclusion (e.g., to determine the truth of this equation 16 + 29 = 15 + 30, the student explained that 16 is 1 more than 15 and 29 is one less than 30, so he reasoned that the quantities on each side of the equation would be the same.)
- Look for patterns (e.g., the pattern when finding two part combinations for a given number, 7 + 0, 6 + 1, 5 + 2, etc.)
- Generalize from examples and make conjectures about the properties of numbers based on those generalizations
- Talk about why things make sense

Connections

- Talk about the connections between the operations of addition and subtraction and addition and multiplication

For specific grade level information on math processes, see the MMSD K-5 Grade Level Mathematics Standards.

Teaching Activities in the Inspecting Equations Block

Inspecting equations activities are divergent in nature and allow for multiple responses, lending themselves to whole class activities. However, all of the activities can be used with a large or small group or as individual practice activities depending upon the choice of number size and the students' levels of proficiency. The activities are listed in order of level of challenge.

GRADE LEVEL
K

Two Part Combinations

What to do:

- ☑ Provide a set of manipulatives and some kind of mat on which to sort the manipulatives.
- ☑ Ask the students to find all the ways to organize the manipulatives into two parts. For instance, the student can have yellow and silver fish. The student finds all the different ways to make a given number using the two colors of fish.

Extension
Students can write equations to record the two part combinations that they find. The equations have the whole number first and the two part combination on the other side of the equal sign. For instance, a student finding two-part combinations for 7 writes 7 = 2 + 5 and so on.



GRADE LEVEL
1-2

Two Part Combinations

What to do:

- ☑ The teacher writes the target number on the board large enough for the entire group to see. He/she asks the students to think of two numbers they can add to make the given number.
- ☑ After a bit of individual thinking time, students are asked to share their thoughts. The teacher makes a list of all the pairs the students share.
- ☑ Students check the accuracy of each pair of numbers.
- ☑ The teacher asks if all possible pairs have been listed and why students think they have or have not determined all possibilities.

Extension:
Increase the possible combinations by allowing other operations to be used.

Expand the possible combinations by Including fractions or negative numbers.

GRADE LEVEL
1-2

True or False Number Sentences (Equations)

What to do:

- ☑ The teacher writes a closed number sentence (equation) large enough for the entire group to see.
- ☑ After several seconds of thinking time, students are asked to indicate whether this number sentence is true, false (not true) or they are not sure. Asking students to put a thumb up, thumb down or to the side and wavering, lets the teacher quickly see the range of ideas.
- ☑ Students share reasons for their decisions, working to convince the others that their thinking makes sense.
- ☑ The teacher writes a new number sentence (equation) below the first, deciding what number relationship or number property relationship to highlight based on the points made during the student discussion. *For example:* A teacher begins by writing a familiar equation such as 3 + 5 = 8. After hearing the students reason why this is a true statement, the teacher writes an unfamiliar equation, in this instance 8 = 3 + 5. The students discuss their thoughts about whether or not this equation is a true statement of equality.

GRADE LEVEL
1-2

Open Number Sentences (Equations)

What to do:

- ☑ The teacher writes an open number sentence (equation) large enough for the entire group to see, e.g., 9 + □ = 10.
- ☑ After several seconds of thinking time, students are asked to share their thoughts about what number could be placed in the box to make the number sentence true.
- ☑ Students share reasons for their decisions, working to convince the others that their thinking makes sense.
- ☑ The teacher writes a new open number sentence (equation) below the first, deciding upon what number relationship or number property relationship to highlight. After discussing the familiar relationship in 9 + □ = 10, the students discuss the next equation the teacher writes, i.e. 9 + □ = 10 + 5.

GRADE LEVEL
2

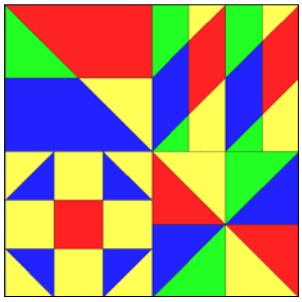
Family of Facts

What to do:

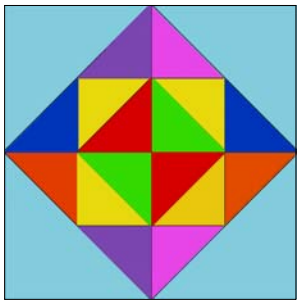
- ☑ The teacher writes three numbers on the board large enough for the entire group to see.
- ☑ These three numbers should be able to be combined with an operation symbol and an equal sign to express a true equality relationship, for instance, 3, 7 and 10 or 10, 5, and 2.
- ☑ After several seconds of thinking time, students are asked to share their thoughts, listing as many true number sentences as they can think of that use these three numbers only one time.
- ☑ Students share reasons for their decisions, working with the others to determine that the number sentences (equations) they have created make sense.
- ☑ The teacher writes a new set of three numbers, deciding upon what number relationship or number property relationship to highlight based on the points made during the student discussion.

For more information:

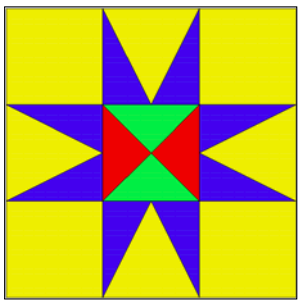
Carpenter, T. P., Franke, M. L. & Levi, L. (2003). *Thinking mathematically: Integrating arithmetic and algebra in elementary school.* Portsmouth, NH: Heinemann.



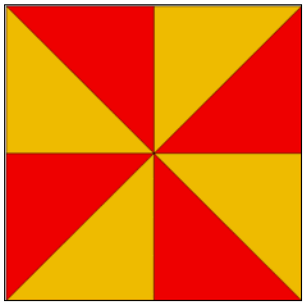
PROBLEM SOLVING



NUMBER WORK



INSPECTING EQUATIONS

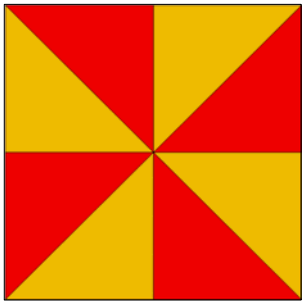


FLUENCY & MAINTENANCE

Chapter 9

Fluency and Maintenance

Learning activities in each block
support the learning activities in the other blocks.



Fluency & Maintenance

K - as needed
1 - about 15 minutes
2 - about 15 minutes

Fluency & Maintenance

Fluency & Maintenance work provides practice over time in order to:

- strengthen understandings of concepts and skills
- reinforce vocabulary
- build efficiency and accuracy

Activities provide experiences to review knowledge, concepts and skills from all the content strands: number, operations, and algebraic relationships; geometry; measurement; and data analysis and probability. Fluency and maintenance work should always be at a student's independent level, determined by teacher observations of daily work, informal teacher created assessments, Fact Interviews and Problem Solving Interviews. (See *Assessment*, Chapter 4.)

The Fluency & Maintenance Block:

- ☑ has activities that use number sizes within a student's independent mental computation level (See Fact Interviews)
- ☑ composes from 5 to 15 minutes of a math hour
- ☑ is most often a small group or individual independent activity
- ☑ can occur while a teacher meets with a small group or an individual for problem solving and number work



The activities must be at each student's independent level.

Teaching Activities for the Fluency & Maintenance Block

Activities for the fluency & maintenance block tend to be convergent, lending themselves most effectively to small group and independent activities. It is important that students are assigned fluency and maintenance activities that are at their independent levels of computation and problem solving skills.

Teachers will find ideas for activities that provide practice to build fluency or to maintain proficiency in the school's curricular resources. Card games are an engaging way to practice using number relationships. The *card game rules* included in this chapter are listed in order of challenge. The following are other suggestions for teaching activities in the fluency & maintenance block:

- pattern block designs with one or two lines of symmetry
- shape puzzles using pattern blocks, tangrams, or pentominoes
- sort, count and graph
- measure with non-standard and standard the length, weight and/or capacity of various objects or pictures of objects
- practice pages
- word finds (geometric and measurement vocabulary)
- fact games on web sites

Card Game Rules

Ten in a Row

Topic: Number Order

Players: 2

Materials: Playing cards (J-K removed)

Object: Sequence cards 1-10 in order.

Play: Each player gets ten cards and lays them face down in a row. The remaining cards are placed in a face-down pile in the middle of the table. The first player draws a card from the deck and replaces a card in his or her row of ten by placing the drawn card face-up in the correct position in the row. The replaced card is turned up and is used to replace another face-down card if possible. Play continues until a space is not available. The card not able to be played begins a face-up discard pile. Players take turns either drawing a new card or using the top card of the discard pile to replace cards in his or her row. The first player to replace all of his or her cards is the winner.

Plus

Topic: Counting on 1, 2, or 3

Players: 2

Materials: Handmade numeral cards (6 each of numbers 1-9) or playing cards (10-K removed)

Object: Collect cards by counting on 1, 2, or 3 mentally.

Play: Choose one "plus" card (1, 2, or 3) from the deck and place it face-up next to the rest of the deck placed face down. The first player turns over the top card and places it next to the "plus" card. The first player states the addition fact, e.g., 6 plus 1 is 7. The second player turns over the card and states the addition fact. When the addition fact is a double (2 + 2, 4 + 4, etc.) the player who turned over the card gets to keep the stack of cards, leaving the "plus" card for further play. Play continues until the deck is gone. The winner has the most cards.

Teaching Tips:

- Adjust the game for each child's mental computation level. Do not go beyond +3 for counting on practice.
- Adapt the game to be a counting back game.
- Adapt the game to be a groups of 2, 5, or 10 game.

Make 10

Topic: Sums of 10

Materials: Handmade numeral cards (6 each of numbers 1-9) or playing cards (remove 10 through king)

Players: 2-4

Object: Collect pairs of cards with sums of 10.

Play: Deal out all cards except the last one that is placed face up in the middle of the table. Each player keeps his/her cards in a face down stack, without looking at them. When his or her turn comes, that player turns over the top card of his stack. If this card can be used with any showing on the table to make 10, the player picks up the cards that make a sum of 10 and sets them aside. If the player can't put two cards together to make 10 he or she places the card face-up with the others on the table and the turn passes to the next player. The winner has the most cards at the end of play.

Fishing for Tens

Topic: Sums of 10

Players: 2-4

Materials: Handmade numeral cards (4 each of numbers 0-9) or playing cards (10-K removed)

Object: Collect pairs of cards with sums of 10.

Play: Deal 8 cards to each player. The players then lay down in front of them all of the pairs that make ten. Players take turns asking for a missing addend to make ten. The pair that makes ten is set aside. If the player asked has the card needed, he or she may ask again for a card from any player. When the player asked doesn't have the card, he or she says, "Go Fish!" and the player draws one card from the draw pile. If the card drawn "makes 10" he or she may continue as the "asker." If not, the player to the left becomes the "asker." Play ends when any player empties his or her hand. The winner has the most that pairs that make 10.

Variations:

- Fishing for Eleven (or 12). Adjust the sets of cards accordingly.
- Players ask for a card only once on a turn.
- Players draw one card at a time from each other's hand. A "wild card" can be added. Players want to avoid ending the game with the "wild card" in hand as in the game "Old Maid."

Mystery 10

Topic: Sums of 10

Players: 2-4

Materials: Handmade numeral cards (6 each of numbers 1-9) or playing cards (10-K removed)

Object: Collect pairs of cards with sums of 10 and determine the value of the hidden cards.

Play: Take two cards fro the shuffled deck and set them aside face down. Do not look at them. The remaining cards are dealt into a 2×5 array between the two players. The top card on each pile is turned up. Players take turns making 10 with a pair of cards. When a card is taken from a pile the next top card is turned up. When a space is vacated, a turned up card can be placed in that spot so that there are always ten cards face-up. Two cards will remain when all cards have been turned up. The hidden cards can be "guessed." Each remaining card has a hidden "partner" that makes ten. If there are no cards remaining, the hidden cards together make a 10, making the guessing very challenging.

Tens Concentration

Topic: Sums of 10

Players: 2-4

Materials: Handmade numeral cards (6 each of numbers 1-9) or playing cards (10-K removed)

Object: Collect pairs of cards with sums of 10.

Play: Place all cards face down in an array of 4 rows with 6 cards in each row. Players take turns turning over two cards to make a sum of 10. If a 10 is made, the player continues. Cards that do not match are turn back face down in the array.

Variation: Change the range of cards to focus on the combinations that students need to practice. For example: Take out the cards above 6 to practice sums to 7.

Dash to the Decade

Topic: Identifying a missing addend under 10.

Materials: Handmade numeral cards (6 each of numbers 1-9) or deck of cards with the 10 through king removed, score sheet (See next page.)

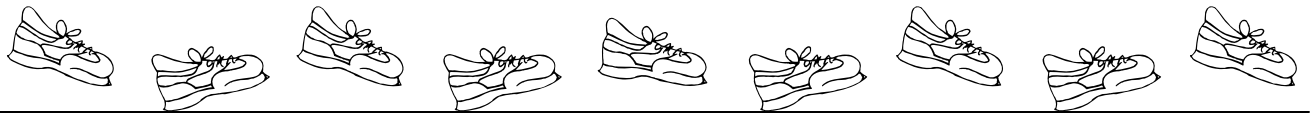
Players: 1 or 2

Object: Complete the "Dash to the Decade" score sheet first or in a shorter time.

Play: Deal entire deck into two piles face down. Players turn over the two top cards from their deck to make a 2-digit number. Record the number on the score sheet and fill in the next "decade" and the amount to reach that decade. Players work at the same time to get to the bottom of the page first. A second way to win is to sum all of the quantities above the arrow to see who had the biggest or the smallest dash.



Dash to the Decade



_____ + → _____

_____ + → _____

_____ + → _____

_____ + → _____

_____ + → _____

_____ + → _____

_____ + → _____

_____ + → _____

_____ + → _____

_____ + → _____

Toss Two Number Cubes

TOPIC: Sums to 12

Players: 1 or 2

Object: Collect data

Play: Roll two number cubes 25 times. On each roll, graph the sum of the two numbers below. Keep track of how many times you rolled each sum on the graph.

Title: **Graph of Sums**

Number of times each sum was rolled

25												
24												
23												
22												
21												
20												
19												
18												
17												
16												
15												
14												
13												
12												
11												
10												
9												
8												
7												
6												
5												
4												
3												
2												
1												
	1	2	3	4	5	6	7	8	9	10	11	12

Sums

Pyramid Solitaire

TOPIC: Sums of 10

Materials: Deck of playing cards 1 - 10

Players: 1

Object: Remove the pyramid.

Set up: 15 cards are dealt face up in the shape of a pyramid in five successive rows (Row 1 – 1 card, Row 2 – 2 cards, Row 3 – 3 cards, etc.). The rows should overlap so that each card is overlapped by 2 cards from the row below. Keep the rest of the cards in a deck and place it face down on the table.

Play: Pick up any pairs of cards in the 5th row that add to 10. A ten may be picked up all by itself. A card in the next row becomes available only when it is not overlapped by another card. Continue until no more pairs can be made. Turn over the top card in the deck and see if you can pair it with a card from the pyramid to make a 10. If you can, remove the cards. If you can't make a 10, turn over the next card. Continue to play until you have made all the 10's possible. If you clear the pyramid of all cards, you win.

Variation: Add face cards and make six rows instead of five for sums to 14 (jack is 11, queen is 12, king is 13.

Salute

TOPIC: Identifying a missing addend or factor

Players: 3

Materials: Handmade numeral cards (6 each of numbers 1-9) or playing cards (10-K: removed)

Object: Be the first player to state the missing addend or factor.

Play; One player serves as the referee. Deal out all cards into two face-down piles in front of the two players. When the referee says, "Go" the players each take their top card and without looking at it, place it on their forehead facing out for the other player to see. The referee states the total and each player tries to be the first to determine what his or her card is based on what number is on the card that the other player is showing.

Variation: Adjust for the facts that students need to practice. For example: Sums to 7 or 15 or multiples under X 5's.

Target : 20

TOPIC: Composing and comparing numbers

Players: 2-3

Materials: Handmade numeral cards (6 each of numbers 1-9), score sheet for every student.

Object: Get as close to 20 as possible.

Play: Deal five cards to each player face up. Take turns adding any three of the 5 cards to make a total as close to 20 as possible. Write the numbers and the total on the Close to 20 Score Sheet. Each player finds his/her score that is the difference between his/her number and 20. Discard the used cards and deal out new cards to replace them. After five rounds, total the scores. The player with the lowest score is the winner.

Variation: Target: 10

Target 20 Score Sheet

Name_____ Date_____

Round 1: _____ + _____ + _____ = _____ _____

Round 2: _____ + _____ + _____ = _____ _____

Round 3: _____ + _____ + _____ = _____ _____

Round 4: _____ + _____ + _____ = _____ _____

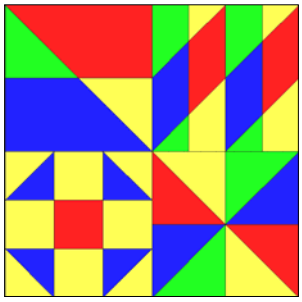
Round 5: _____ + _____ + _____ = _____ _____

Total Score: _____

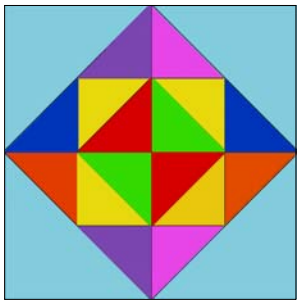
For more information

Dana, M. (1998). *Math Practice Games.* Grand Rapids, MI: Instructional Fair, Inc. (Blackline resource for Grades 1, 2, 3, 4, and 5 available from a math resource teacher)

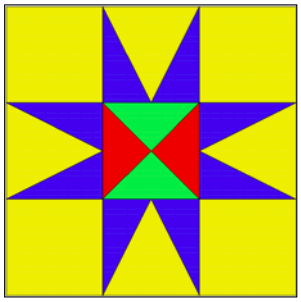
1	2	3
4	5	6
7	8	9



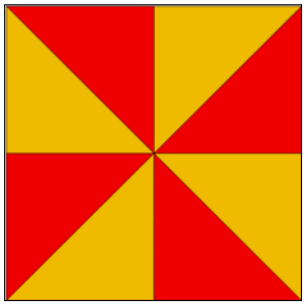
PROBLEM SOLVING



NUMBER WORK



INSPECTING EQUATIONS



FLUENCY & MAINTENANCE

Chapter 10

Intervention

This work was made possible through the generous support of....

The Madison Community Foundation that understood the importance of all children exiting first grade as proficient math students and provided the funds to secure supplies, the services of a consultant and release time for first grade teachers throughout the district.

DiME, Diversity in Mathematics Education Project, a National Science Foundation Center for Teaching and Learning Project, that provided the funds to release a team of first grade teachers from their classrooms to develop this intervention project.

SCALE, Systemwide Change for All Learners and Educators, a National Science Foundation Math Science Partnership Project that provided support through the WCER for evaluating the effectiveness of the professional development.

Angela Andrews, our consultant from National Louis University whose expertise, ideas, and knowledge guided us as we developed this intervention initiative.

Barbara Marten, our writer who synthesized, organized and simplified all of the information regarding first grade intervention into a concise, easy to read and essential chapter.

And most of all,
The 40 elementary teachers who were eager to learn how to accelerate the development of mathematical understanding of students who were lagging behind, conscientiously tried out and provided feedback on strategies, and provided expertise on how to embed this individual or small group instruction into the classroom experience.



Every Child Counts/Accounting for Every Child

The Goal

The goal of this intervention initiative is to provide teachers with the information and materials necessary to help primary grade students become proficient in math and ready to benefit from the grade level math instruction. Only with the teachers' deft scaffolding of the presentation and practice activities for new skills and concepts will the students be able to make a significant shift in their thinking and exit with grade level understandings in math.

To address specific student learning needs, teachers need to:

- find out what each child already knows, at what level each is in his/her understanding of mathematics
- understand what each child needs to know in order to progress to the next level,
- know ways to help each child learn what he/she needs to know in order to progress to the next level
- implement an appropriate sequence of learning activities

This chapter and the accompanying professional development will provide teachers with assessments, a sequence of number development and the corresponding learning activities to address counting, number identification, grouping and chunking of numbers and story problem solving.

The Process

If a student's Fall PMA proficiency level is a 1 or 2, the teacher should use the Number Development Assessment, the Problem Type Interview and the Addition Fact Interview to learn more about the student's math understanding.

These assessments reveal a student's understanding of:

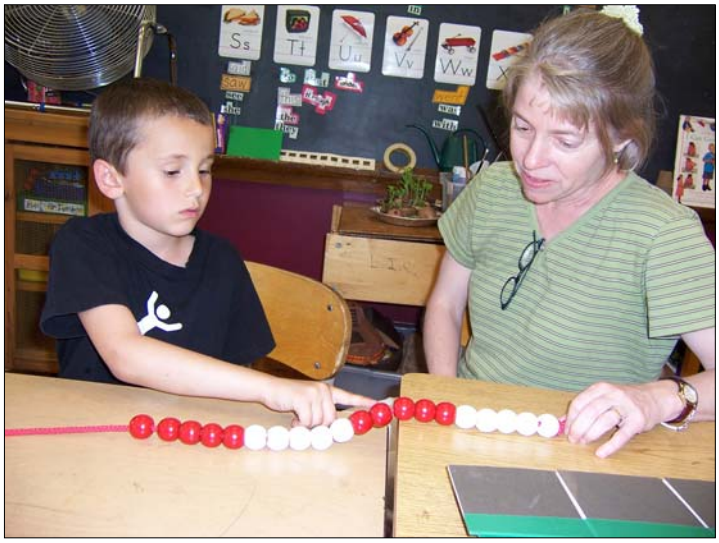
- forward counting
- number directly after
- backward counting
- number directly before
- number identification
- sequencing numbers
- counting on and counting back from strategies
- decomposing and composing numbers
- problem solving strategies for CGI problem types
- addition fact fluency

Teachers can then use the chart on pages 166-168 and the activities described on pages 174-193 to locate appropriate tasks and number ranges for instruction.

Teachers will need to re-administer the assessments periodically throughout the school year to guide decision-making regarding next steps for the student.

When can this happen?

Some teachers have found time during the school day to provide intervention instruction by pulling students in small groups or individually during the problem solving block, the number work block or the fluency and maintenance block. Other teachers have used daily routine activities and independent work time to meet with individual students or small groups of students on math at their level.



The Contents of This Chapter:

- Stumbling Blocks
- Activities to Support Counting and Number Identification
- Activities to Support the Development of Counting On and Counting Back From Strategies
- Activities to Support Composing and Decomposing Numbers
- Activities to Support Problem Solving
- Number Development Assessment and Materials
- Planning and Progress Record
- Number Cards
- Arrow Cards
- Ten Frame Dot Cards
- Supply List

Sample activities/worksheets developed by the study teachers can be found on the http://dww.madison.k12.wi.us

Guidelines for Implementing a Math Intervention Initiative in Number at the Primary Levels

(Teachers may need to be flexible in the use of these guidelines because a student's number development may not match all the characteristics of a specific level.)

What the Child Knows at the Beginning of the Level	What the Child Needs to Learn To Progress to the Next Level			
	Counting and Number Identification	**Counting On and Counting From**	**Composing/Decomposing Numbers**	**Problem Solving**
Level A: • Can recite numbers (rote count) to 10 but does not have 1 to 1 correspondence	• Forward counting 1-20 from any number • Backward counting 10-1 from any number • Identifying numbers 1-20 • Sequencing numbers 1-20 • Naming the number directly after (within 1-20) • Naming the number directly before (within 1- 5) • Counting sets up to 20 with 1:1 correspondence		• Creating and identifying finger patterns to 5 • Identifying regular dot patterns to 6 • Matching dot patterns and finger patterns (within 1-5)	• Solving problems embedded in daily routines using the direct modeling strategy: ▪ Join Result Unknown, (JRU), ▪ Separate Result Unknown (SRU), ▪ Multiplication (M) and ▪ Measurement Division (MD)

Guidelines for Implementing a Math Intervention Initiative in Number at the Primary Level

(Teachers may need to be flexible in the use of these guidelines because a student's number development may not match all the characteristics of a specific level.)

What the Child Knows at the Beginning of the Level	What the Child Needs to Learn To Progress to the Next Level			
	Counting and Number Identification	**Counting On and Counting From**	**Composing/Decomposing Numbers**	**Problem Solving**
Level B: • Can forward count to 20 • Can backward count from 10 • Can name number directly after for numbers 1-20 but may have to count from 1 • Can name number before with numbers1-5 but may have to count from 1 • Can identify numbers 1-20 • Can subitize regular dot patterns to 6 • Can count out sets up to 20 with 1:1 correspondence • Can solve JRU, SRU, M and MD story problems that occur in daily routines	• Forward counting 1-30 starting from any number * • Backward counting 30-1 starting from any number * • Identifying numbers 1-30 • Sequencing numbers to 30 • Naming up to 3 numbers directly after a given number (1-30) * • Naming up to 3 numbers directly before a given number (1-30) *	• Determining the total quantity of 1 or 2 sets when at least one set is hidden from view	• Using finger patterns to represent numbers 5-10 • Using pairs of dots to determine quantity (within 1-10) • Using irregular dot cards to determine quantities under 10 • Ways to Make a Number	• Solving JRU, SRU, M and MD story problems with numbers to 30 using direct modeling (contexts progressing to increasingly removed situations)

*Prerequisite skills for counting on and counting from

Guidelines for Implementing a Math Intervention Initiative in Number at the Primary Level

(Teachers may need to be flexible in the use of these guidelines because a student's number development may not match all the characteristics of a specific level.)

What the Child Knows at the Beginning of the Level	What the Child Needs to Learn To Progress to the Next Level			
	Counting and Number Identification	**Counting On and Counting From**	**Composing/Decomposing Numbers**	**Problem Solving**
Level C: • Can count forward 1-30 starting from any number • Can consistently count backward 10-1 starting from any number • Can inconsistently count backward 30-0 • Can identify all single digit numbers • Can identify numbers 20-29 • Can inconsistently identify numbers 11-19 • Can inconsistently sequence numbers to 30 • Can name number directly after 1- 30 • Can inconsistently name number directly before 1-30 but may have to count up from a smaller number • Can subitize finger patterns to 10 • Can use pairs of dots to determine quantity • Can determine total quantity of two covered sets but may count from 1 • Can solve JRU, SRU, M and MD story problems using direct modeling	• Forward counting 1-100 (starting from any number) * • Backward counting 30-0 (starting from any number) * • Naming up to 3 numbers directly after (1-100) * • Naming up to 3 numbers directly before (1-30) * • Identifying numbers to 100 • Sequencing numbers to 100	• Determining the number of counters in two covered sets without counting from 1 • Determining the number of counters in a covered set after 1, 2 or 3 have been removed	• Composing and decomposing numbers to 10 • Subitizing quantities to 10	Solving: • M and MD story problems using direct modeling • JRU (initial set is from 1-30, second set is no more than 3) using a "counting on from first number" strategy • SRU (initial set is 1-10, removed quantity is no more than 3) using a "counting back from" strategy

Stumbling Blocks

Focus

Some students have difficulty identifying the main purpose of the lesson and/or staying focused on the lesson for even short periods of time. These students may need to be actively involved during the lesson. The teacher might use additional attention-getting strategies such as: calling the student's name before asking a question, reminding the student to pay attention, changing the tone of his /her voice, and/or providing an engaging context for the activity.

Language

There may be a delay in the language development of some students. Their receptive and expressive language may be immature. Their vocabulary may be lacking. The teacher may need to simplify his/her language, introduce new vocabulary slowly, break down the concept/skill into small steps with instructional scaffolding, provide the students with extra practice, and use highly visual models such as the bead string to illustrate concepts. The teacher can ask the student to repeat the request and/or the concept as a way to check for the student's understanding.

Making Connections

Some students may need help in making connections. For example, seeing the relationship between doubles and doubles plus/minus one may not be apparent to them. They may need opportunities using various activities and tools to attend to the relationship. They may need the language used to express the relationship modeled by the teacher or they may need to explain the relationship in their own words.

Memory

Students may need more opportunities to practice concepts and skills in order to more firmly commit them to memory. Providing students with different activities and tools to practice a concept or skill keeps them engaged for a longer period of time extending their ability to remember. Emphasis should be on helping the student understand the meaning of the lesson to facilitate recall rather than using rote drill.

Speech

Some students make articulation errors, such as:

- sound substitutions: "f" for "th" so thirty becomes firty; "d" for "t" so forty becomes fordy
- omitting final sounds/syllables: eightee instead of eighteen so there is no auditory difference between eighty and eighteen

Modeling correct pronunciation and exaggerating sounds sometimes helps students begin to hear the differences. For example, the "Australian" pronunciation exaggerates the "t" in 4T, 8T, etc. and substitutes "Deen" for teen numbers, 4Deen, 8Deen, etc.

Visual Discrimination of Numbers

Students may:

- write digits backward, but read them correctly
- reverse 2-digit numbers, confusing 78 and 87, 12 and 21, 18 and 81, etc.
- make visual approximations or top/bottom flips, confusing 6 and 9, 3 and 8, 6 and 8, 2 and 5, etc.

This difficulty causes some students to incorrectly name numbers they are using. For example, they may solve the problem "What comes after 56" by counting on from 59 and thus arriving at 60 as an answer. The process the student used was correct; the answer was not. These students may need extra practice with the number track, counting on a number line, using 100 cards, using arrow cards in combination with a 100 bead counting frame, bingo games, and working in the 3-digit number range.

Wait Time

Some students may need more wait time before responding to a question or request. Success for these students is often dependent upon allowing them adequate wait time to process their thinking.

Work Time

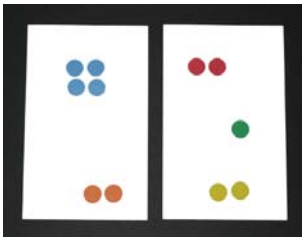
Some students may work at a slower rate than others. To accommodate a slower work rate, modifications might include fewer problems on a page, fewer pages of problems, and more time to complete assigned work other than recess and free time. Work given during the fluency and maintenance block should always be within the student's independent mental computation level.

Tools and Their Uses

When choosing the activities and tools for a particular lesson, the teacher may want to keep in mind that some students need the concept presented and practiced using a variety of activities and tools. Teachers can help students gain conceptual understanding by helping them make connections.

Covered Tasks

In a covered task, the teacher covers the counters he/she uses when presenting a problem to a student. The first set of counters may be covered after showing the student the set. Initially, more counters may be added to the set or some may be taken away while the student is looking to create the problem for the student to solve. Or the second set of numbers may be placed under a second cover. Later, the sets may be increased or decreased while the student is not looking. The purpose for covering the counters is to encourage the student to use a visual image of quantities which can help develop the counting on strategy.



Dot Cards

Dot cards come in a variety of forms. For example, dots can be arranged as ten frames, domino patterns, or dice patterns. They also can be arranged regularly and irregularly on cards. Dot cards provide a vehicle for helping students develop the ability to recognize the quantity of a small group of objects without counting. This is called subitizing. Teachers can use the student's ability to subitize to support his/her understanding of addition and subtraction.

There are two basic types of subitizing: perceptual and conceptual.

Perceptual subitizing is recognizing a number without using a mathematical process. A set of three objects is recognized immediately as 3 without counting each object.

Conceptual subitizing is a more advanced skill. It is recognizing at the same time the number pattern as a composite of parts and also as a whole. For instance, when seeing a domino like  a child who is conceptually subitizing will see each half of the domino as a set of 3 dots and the whole domino as two groups of 3 for 6 dots.

Subitizing is a skill that benefits from controlled developmental practice. It is best to begin with simple forms such as groups of dots

or circles with regular arrangements (rather than pictures of objects or a mixture of shapes), symmetry, and good figure/ground contrast. Playing games that use dice or dominoes encourages quick recognition of number pattern. Providing activities that use temporal and kinesthetic, rhythmic, and spatial-auditory patterns also help students develop subitizing skills.

The Empty Number Line
An empty number line is a number line without printed numbers. It provides students with a way to visualize number relationships and to represent their thinking. See page 110.

Number Cards
Number cards are numbers written on cards about 2.5 by 3 inches. Because they can be used for many tasks, several sorted stacks are desirable. For example, stacks of numbers- 1-10, 1-100, 5-15, 25-35- make it easier for the teacher to get the numbers most appropriate for a particular student.



The Number Track
A number track is a rectangular cardboard strip divided into 5 equal sections, each with a covering flap. Each track contains five consecutive numbers. A set of six tracks makes up the number line 1 to 30. Tracks can be placed together as the student's knowledge of number expands. A seventh blank track using post-its with numbers on them, for example, can be used to extend the student's practice with consecutive 3-digit numbers.

Students use the number tracks to practice counting forward and backward, number identification, naming the number after and the number before, and counting on and counting back from to solve result unknown problems.



The Rekenrek or Arithmetic Rack
A Rekenrek is composed of at least 1 rod with 10-20 beads. The beads are organized into groups of 5 (or10) red and 5 (or 10) white. Beads are stored at the end of the rack that is to the student's right. The student solves the problem by pushing the beads to his/her left and reading the beads left to right.

Teachers use the Rekenrak to support the student's learning of doubles, decomposition of numbers and privileging 5 and 10.



Sliding Card
The sliding card is made by folding a square piece of oak tag, cutting a smaller square in the middle of one half, and stapling closed the bottom of the side opposite the fold. Numbers in sequence are written on strips that can be pulled through the

window frame. Forward and backward number sequences, including the next number after or before, can be practiced and verified by students.

Ten Frame Cards

Ten frame cards are two rows or columns of five squares. Dots are placed consecutively in the squares to represent a number. They can be arranged pair wise or five wise. Ten frame cards can be used to help students conceptualize that there are two groups of 5 spaces, recognize the spatial configurations for each number, and privilege 5 and 10 when completing additive and subtractive tasks. (See page 221 for examples of pair wise and five wise ten frame cards.)



The 20 Bead String

There are 20 beads on a string, alternating 5 red and 5 white. This calls attention to groups of 5 and the relationship of numbers 5 and 10. For example, 4 is 1 away from 5; 9 is 1 bead less than 10. This action is called privileging 5 or 10. Beads are stored on the end of the string that is to the student's right. The selected beads, those being used in the problem, are at the end of the string that is to the student's left. Always begin counting with the first set of red beads.



Two-Colored Counters

There are various two-colored counters available commercially. Round, flat discs with surfaces of two contrasting colors are desirable. They can be easily slid under covers for covered tasks.



Activities to Support Counting and Number Identification

Using knowledge of a student's counting skills, teachers can then choose a range of 5-7 numbers for instruction in forward/backward counting, naming the numbers before and after, number identification, and sequencing. The range should begin with a known number and proceed to the unknown.

Forward Counting and Naming the Number Directly After

Activity: **Classroom Routines**

Tools: Routine Dependent

Directions: Practice counting forward during daily routines, such as: counting the children who want hot lunch and those who want cold lunch; calendar activities, how many days have been sunny/cloudy, how many days students have been in school, etc.

Extension: Count by 2s, 5s, and 10s (both on and off the decade) accompanied by visual models such as base ten blocks, ten frames, rekenrek, empty number line, 100s chart, sliding chart and arrow language.



Activity: **Counting Collections**

Tools: Choice of Counters

The choice of counters may motivate children to count. Teachers might provide counters related to science units (insects, snowmen, zoo animals, etc.) or counters that are fun (vehicles, shoes, etc.).

Tasks: To practice counting give each student a bag of counters. The number of counters can be either at the student's independent or instructional level. The student can count the entire set or sort the set into-sub sets and count them. For example, if the student is working on counting 1-10, the bag might include 6 pick-up trucks, 9 vans, 8 sedans, and 10 racecars. The student sorts the vehicles and places each group in a column on 1-inch graph paper. Each student could share his/her information with the group. The teacher might record group information on a large graph, helping the students count beyond 10.

Activity: **Forward Counting Songs and Rhymes**

Directions: See pages 247-253 for examples of counting songs and rhymes. Additional songs and rhymes are included on the http://dww.madison.k12.wi.us .

Activity: **Forward Counting with the Bead String**

Tools: The 20 Bead String

Directions: See page 254-255 for using the 20 bead string to facilitate forward counting.

Activity: **Forward Counting on the Number Tracks**

Tools: Number Tracks

Directions: To practice counting forward from numbers other than 1, place number tracks 6-10 and 11-15 end to end. Close all of the flaps. The teacher or student opens any flap. The student then counts forward from that number. Students can check their accuracy by opening the flaps. Open other flaps to provide more practice. Line up other number tracks when a student is ready for numbers beyond 15.

Extension: Use numbers over 30 printed on post it notes under the flaps to extend the counting practice to 100

Activity: **Number Track After**

Tools: Number Track

Directions: To practice identifying the number directly after, the teacher or student opens any flap. The student says the number directly after. Students can check their accuracy by opening the next flap. As students have success identifying the number directly after with numbers 1-10, move on to numbers 11-20.



Activity: **Sliding Forward**

Tools: Sliding Card

Directions: The teacher positions the number line in the card opening. As the student counts forward, the numbers are revealed in the opening by sliding the number line through the card so the student can confirm or correct answers.

Extension: Modify the sliding card so the number line moves vertically. Use a 100s chart cut vertically to practice counting by 10 off the decade. (ex. 4, 14, 24, 34…)

Activity: **Sliding Card After**

Tools: Sliding Card

Directions: The teacher positions the number line in the card opening. The student names the number directly after the number shown in the window. The number is revealed in the opening by sliding the number line through the card so the student can confirm or correct the answer.

Backward Counting and Naming the Number Directly Before

Activity: **Counting Backward on the Number Track**

Tools: Number Track

Directions: To practice counting backward, follow the process described for counting forward using the number track except have the student count backwards from the exposed number.

Activity: **Number Track Before**

Tools: Number Track

Directions: To practice naming the number directly before, follow the process described for naming the number directly after using the number track except have the student say the number before.

Activity: **Backward Counting Songs and Rhymes**

Directions: See pages 247-253 for examples of counting songs and rhymes and the http://dww.madison.k12.wi.us.

Activity: **Sliding Backward**

Tools: Sliding Card

Directions: The teacher positions the number line in the card opening. As the student counts backward, the numbers are revealed in the opening by sliding the number line through the card so the student can confirm or correct answers.

Activity: **Sliding Card Before**

Tools: Sliding Card

Directions: The teacher positions the number line in the card opening. The student names the number directly before the number shown in the window. The number is revealed in the opening by sliding the number line through the card so the student can confirm or correct the answer.



Number Identification

Activity: **What's This Number?**

Tools: Number Track

Directions: Line up the number tracks containing the numbers appropriate for the student. Close all flaps. The teacher opens any flap and asks, "What is this number?" If the student has difficulty identifying the number, the teacher opens the preceding flap and asks the student again. If the student continues to have difficulty, the teacher opens the first number in the track and asks, "Does this give you a clue?" If the student still has difficulty, the teacher identifies the first number in the track and asks, "Does this help a bit?"

Activity: **Number Turn Over Game**

Tools: Number Cards 1-10, spinner or 2 dice

Directions: Two students and an adult can play this game. Each student is given five cards from a deck of number cards 1-10. Students place their cards face up on the table. The teacher or other adult spins a spinner or tosses 2 dice and announces the number name. Whoever has the card with that number on it turns it over. The goal is for each player to turn all his/her cards over.

Extension: Use number cards 1-20



The teen numbers are difficult for some students to learn. The teacher can modify the game by limiting the number of cards in the deck, allowing more practice with the teen numbers. The goal is to have all numbers placed in the "win" pile on the first attempt.

Activity: **Name It to Claim It Game**

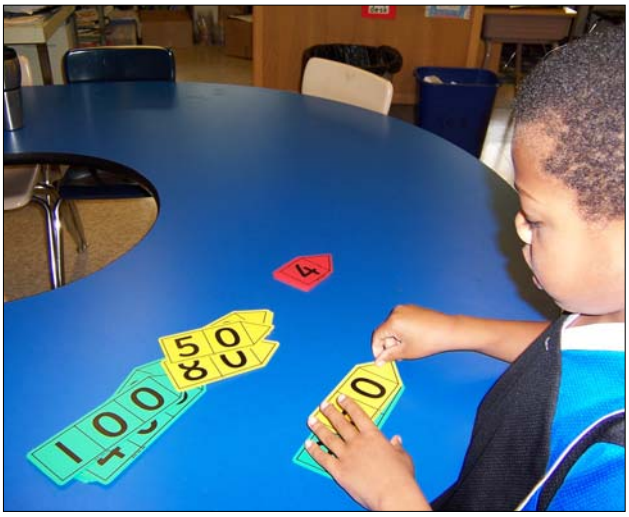
Tools: Number Cards 1-100

Directions: One student plays this game with one adult. The adult shows a number to the student. If he/she can name it, it goes in his/her "win" pile. If the student cannot name it, the adult names it for the student and then puts it in the "2nd chance" pile. Once all of the cards have been shown, the adult shows the cards from the "2nd chance" pile again. Numbers correctly identified are placed in the student's "win" pile and incorrectly identified numbers are named by the adult and placed in the "2nd chance" pile.

Activity: **Number Building**

Tools: Arrow Cards, 2 sets each of decades and ones

Directions: Each student turns over the top card from each pile and names the 2-digit number. For instance, if a 60 and a 5 are turned over and the student says 65, she/he keeps both arrow cards. If the student misnames the number, both cards are placed in the "2nd chance" pile. The teacher helps the student position the arrow card correctly and asks the student, "Does this help?" If the student still is challenged, change the arrow cards to make smaller numbers.



Activity: **Number Card Match**

Tools: Number Cards, 2 sets of teen numbers and 2 sets of "ty" numbers (A "ty" number is a number ending in zero, such as 20, 40, etc.)

Directions: The teacher places the teen cards face down in one row and the "ty" cards face down in another row. Student turns over two cards in the same row. If the two cards match, the student says, for example, "This is 14 and this is 14. Match!" If the 2 cards do not match, the student says, for example, "This is 40 and this is not 40. No match."

Extension: Use the same two sets of cards, but mix up the two rows.

Activity: **Make It Disappear!**

Tools: Dry erase board, marker, eraser

Directions: To practice identification of numbers that are easily confused, the teacher or adult writes a number that the student is currently confusing with others, such as 15, 50, and 51. After writing the number, the adult asks, "Is this 15?" If the student replies correctly saying, "Yes, it is 15", the student gets a point and erases the number. If the student is incorrect, the adult gets the point.

Activity: **Number Sort**

Tools: Number Cards of two easily confused numbers, for example 12 and 21 (there should be a total of 16 to 20 cards in all with one number on half of the cards and the other number on half), a sorting board (this can be a dry erase board, a piece of cardboard, etc.) with an illustration of the 2 numbers to be sorted at the top (for example dot cards or ten frame cards}.

Directions: The student sorts the cards by number.

Extensions: 1. The teacher mixes up the cards and lays them out, face up on the table. Then he/she asks the student to find all the 12s, for example, and give them to him/her.

2. The teacher mixes up the cards and puts them in a straight line on the table. He/she then asks the student to name each number, reading left to right.

3. For independent activity see DWW.

Activity: **Rekenrek How Many?**

Tools: The Rekenrek, Number Cards 1-20

Directions: The teacher holds up a number card, 10 or less, and asks the student to show that many beads with one push. After the student has learned to push over 10 beads with one push, the teacher can demonstrate how to show teen numbers with 2 pushes. The activity continues with the teacher holding up a teen number card and the student showing that many beads with two pushes.

Extension: Use a 100 bead/ten bar rekenrek or a 100 bead/one bar rekenrek to model numbers greater than 20.

Activity: **Bead String How Many?**

Tools: The 20 Bead String

Directions: See page 254-255 for ways to use the 20 Bead String to foster number identification.

Sequencing Numbers

Activity: **Ten in a Row Card Game**

Tools: Number Cards 1-10

Directions: See page 152 for directions.

Extension: The game can be expanded to include Number Cards, 1-20.

Activity: **Order Them!**

Tools: Number Cards (1-100)

Directions: The teacher chooses an appropriate series of numbers from the deck (for example 37-44), mixes them up, and places them face up on the table. The student places them in order from least to greatest and then names each number.

Modifications:

1. Choose fewer cards (only 4-5)
2. Choose a series that does not cross a decade

Extension: Choose numbers that are not in sequence directly after one another, for example 42, 61, 75

Activity: **Repair the Number Line!**

Tools: 3 different sequences of numbers cut from the number line

Directions: The student replaces the sequences of numbers to complete the number line.

Extension: Use a hundreds chart that has been cut apart instead of the number line.



Activities to Support the Development of One to One Correspondence, Counting On and Counting Back From

One to One Correspondence

Activity: **Organizational Maps**

Tools: Two part place mats with each part a different color, blank ten frames, counters

Directions: Organizational maps provide a support for the student to "see" which counters have been counted and which need to be counted. In the case of the two part place mat, the student moves a counter from one side of the mat to the other as the count is made. When a blank ten frame is used, the student fills a ten frame with one counter per square as the count is made. Begin with quantities within the perceptual subitizing range (<6) and ask the student to double check using subitizing.

Activity: **Polka Dot Matching**

Tools: Dot cards, counters

Directions: Quickly flash a small quantity of dots or objects within the perceptual subitizing range. Ask the student to recreate that small quantity using a similar counter. The student compares the two quantities to determine a match

Counting On

Activity: **Counting On with the Bead String**

Tools: 20 Bead string

Directions: See pages 254-255 for ways to use the 20 Bead string to foster counting on.

Activity: **Emergent Covered Tasks**

Tools: Two-colored Counters and one Cover

Directions:

- The teacher places 4 or fewer counters in an array for the student to see.
- The teacher asks, "How many counters are there?"

- If the student answers without counting, the teacher says, "You just knew that answer! You didn't have to count!" (If the student counted from one, start with 1 counter under the cover and repeat the process. Increase the number of counters under the cover by one.)
- Next the teacher covers the set of 4 or fewer counters. While the student is watching, the teacher slides another counter of a different color under the cover.
- The student is then asked how many are underneath the cover now. If the student responds without counting from one, the teacher says, "You didn't have to count to know that! How did you figure it out?"
- The student who counts on to solve this problem may say that one was added to the others so now there is one more.
- The teacher can expand on what the student said by saying, "You just knew that if there was one more counter, then the total number of counters is the next bigger number."
- If the student is not able to describe his or her thinking, the teacher can say, for example, "I once had a student who knew that if there was one more counter, the total number of counters under the cover would be the next bigger number. Is that how you solved the problem?"
- If the student counted on, repeat the process, each time beginning with one more counter underneath the cover (5 + 1, 6 + 1, 8 + 1, etc.).
- When the student is successful with problems starting with a set more than 4, the teacher can add two and then three more counters to the original covered set.

Activity: **Developing Covered Tasks**

Tools: Two-colored Counters and two Covers

Directions: Repeat the process described above except put the second set of counters under a separate cover. After having the student describes how he or she solved the problem, the teacher restates the student's actions in terms of counting on.

Activity: **Counting On with Number Cards and Covered Tasks**

Tools: Number Cards (1-100), base ten blocks, ten frame cards, counters, two covers

Directions: Teacher shows the student a number card (ten frame or base ten blocks) that is within his/her forward counting level and covers it. While the student is watching, the teacher places 1, 2, or 3 counters under the second cover and asks for the total of both sets. After the student has solved the problem, the teacher asks how he/she solved it. The teacher then restates the student's actions in terms of counting on.

Extension: As the student's forward counting ability grows, the teacher can expand the numbers used. When the student is able to cross decades, the teacher can introduce problems that cross the decade, such as 29 and 2 counters.

Activity: **How Many More?**

Tools: Number Tracks appropriate to number range

Directions: The teacher presents a problem such as 9 + 2. Then he/she opens the 9 flap and asks the student "How much is 2 more?" After responding, the student opens the flap to check his/her answer.

Counting Back From

Activity: **Counting Back From with Covered Tasks**

Tools: Two-colored Counters and one Cover

Directions:

- The teacher shows the student a set of 4 or fewer counters
- The teacher covers the set and removes one counter.
- The teacher asks the student "How many counters are left under the cover?"
- The student who counted back to solve this problem may say "One was removed from the set so now it's just the next number down".
- The teacher can expand on what the student said by saying, "You knew that if there was one less counter, then you could just count back one."
- The original set can increase gradually within the student's backward counting range with 1, 2 or 3 counters removed at a time.

Activity: **Counting Back From on the Number Track**

Tools: Number Tracks appropriate to number range

Directions: The teacher presents a problem such as 9-2. Then he/she opens the 9 flap and asks the student "How many is 2 less?" After responding, the student opens the flap to check his/her answer.

Activity: **Counting Back From on the Empty Number Line**

Tools: Empty Number Line

Directions: See page 110 for ways to use the empty number line to represent counting back.

Activities to Support Composing and Decomposing Numbers



Show the cards in random order for 1-2 seconds. Observe how the student answers. Does he/she seem to guess, answer quickly with certainty, reproduce the pattern physically with fingers, nod his/her head while silently counting, etc? The teacher may want to have a conversation with the student about how he/she knew the total number of dots.

Activity: **Look Quick!**

Tools: Dot Cards with up to six black dots in dice formation

Directions: The teacher says, "I am going to show you a card with some dots on it. I want you to tell me how many dots are on the card. You will have to be very quick because I am going to show the cards very fast and then hide them. Are you ready?"

Extensions: Use dot cards with regular and irregular dot patterns greater than 6. Each cluster of dots within the larger quantity can be a unique color. This can support the child in building fact fluency.

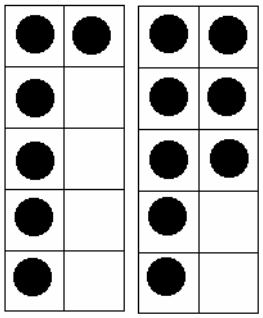


Activity: **Which Card Does Not Belong?**

Tools: Sets of Irregular Dot Cards with total number of dots less than 10 (with each quantity represented by 3 different irregular patterns), a Playing Mat (with numbered spaces for 4 dot cards), a Cover (to easily cover the dot cards)

Directions: The teacher places 4 irregular dot pattern cards on the playing mat, 3 of which have the same number of dots and 1 of which has a different number of dots. Allow the student to see the display for about 5 seconds and then cover all of the cards. After the teacher asks the student, "Which card does not belong?" and the student responds, the teacher and student should discuss how the student chose the card. The teacher can help the student see that the dots could be seen in groups or chunks, thus helping them to see the total quickly.

Modification: The teacher shows just two dot cards and asks which card has more dots.



Activity: **Building Five and Ten**

Tools: Ten Frames Cards (with dots on the cards to represent numbers 1-10)

Directions: Begin with ten frame cards representing quantities five or fewer. Show the student the card for 1-2 seconds and then hide it. Ask the student to identify the quantity. Conversation can highlight the relationship of the quantity to five." For example: "I knew it was 3 because there were two boxes missing dots." As the student progresses, use ten frame cards with quantities from 6-10. The child can identify the quantity and discussion can note the relationship of the quantity to both five and ten.

Extension: Show the student a full ten frame and a partially filled ten frame. Ask the student to indicate the total quantity and then show how the two ten frames represented the total using an empty number line.

Activity: **How Many on the Rekenrek?**

Tools: The Rekenrek

Directions:



Assumption: The Rekenrek has been introduced to the student.

- The teacher says, "We're going to play a game with this Rekenrek. I am going to show you some of the beads, and you have to tell me how many you see." The teacher then pushes over 2 beads and asks, "How many?" The beads are returned to the student's right. Then the teacher pushes over 4 beads and asks, "How many?"
- Next the teacher pushes over 6 beads and covers the Rekenrek. After asking, "How many" and the student responds, the teacher asks the student to explain how he/she knew, reinforcing the explanation that included subitizing. The student might say, "I knew there were 5 red ones so when I saw 1 more white on I knew it was 6."
- The teacher might continue by pushing over 9 beads and asking, "How many? How did you know?" The teacher should reinforce the explanation that includes privileging 10. Such an explanation might be, "I knew there were 10 beads, so if there is just one less than 10, that is 9. Another student might privilege 5 saying, "I knew there were 5 white beads, and I only saw 4. I know 5 + 5 is 10, so 5 + 4 is 9. The teacher should reinforce the non-counting strategies.



The one-push rule discourages counting from 1 and encourages subitizing five and privileging 5 and 10. These strategies help the student internalize such facts as 7 is 5 and 2 more and 9 is 1 less than 10.

Activity: **One Push**

Tools: The Rekenrek

Directions: The teacher directs the student to show a specific number of beads on the Rekenrek in just one push. For instance, to show 7, the student would push over 5 reds and 2 whites in one push. (The student can use either the top or bottom row of beads.) The teacher continues with specific numbers, encouraging the student to make the number with just one push.

Activity: **Composing and Decomposing Five**

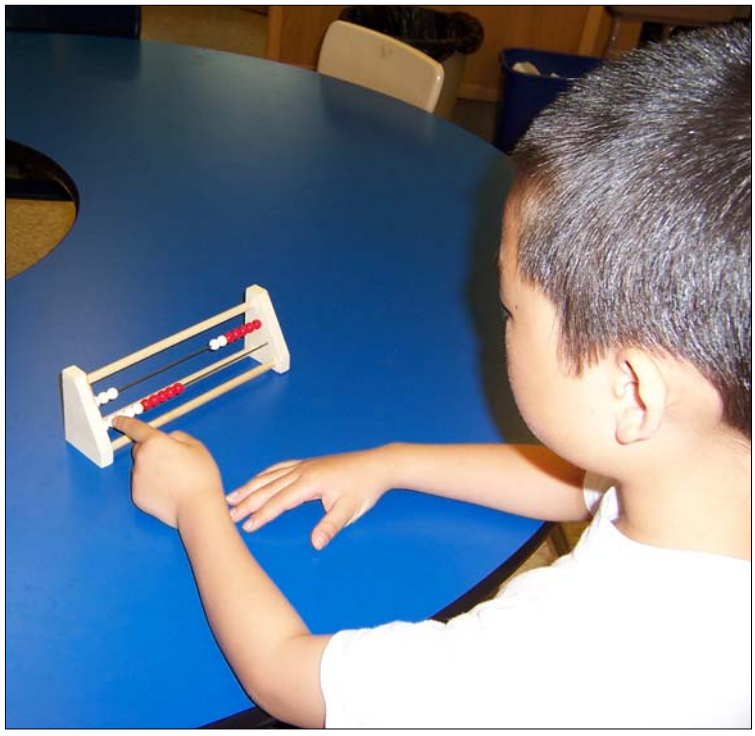
Tools: The Rekenrek (use top row of beads only), and chart paper

Directions: The teacher starts with all of the red beads to the student's left and all of the white beads to the student's right. He/she says, "For this game, I want you to push some red beads and some white beads to the middle. The rule is that you have to have 5 beads in the middle when you are finished. I will record your combinations on this chart."

The teacher records the combinations as the student gives them, but chooses where on the chart she/he records the combination so that the chart gives a visual pattern of ways to make 5. The chart should look similar to this:

1 red + 4 whites = 5 beads
2 red + 3 whites = 5 beads
3 red + 2 whites = 5 beads
4 red + 1 white = 5 beads

Extension: Follow the same procedure for other numbers 1-10.



Activity: **Composing and Decomposing Doubles**

Tools: The Rekenrek, Chart Paper

Directions: The teacher can use the rekenrek to introduce doubles. The teacher begins by separating a group of red beads to one side of the rekenrek and a group of white beads to the other side. Then she or he pushes 1 white bead and 1 red bead to the center. The teacher might record the number sentence, 1 + 1 = 2 or have a student write it on the board. The teacher continues by pushing 1 more white bead and 1 more red bead to the center and recording the number sentence until all doubles through 5 + 5 = 10 have been shown.

Activity: **Doubles Plus One and Minus One**

Tools: The Rekenrek and a Popsicle Stick or another manipulative to divide like a pencil

Directions: Once a student is familiar with doubles facts, the teacher can help him/her develop strategies for using his/her knowledge of doubles to solve problems involving doubles + or – The teacher pushes 3 red beads to the center and 2 red beads to the center from the other side. Using a popsicle stick or other marker, the teacher points out the place on the Rekenrek where the 2 + 2 is. Then she/he says, "If you know that 2 + 2 is 4, then can you figure out what 3 + 2 is?" Continue this activity with similar problems using doubles.



Prerequisite skill: counting forward from any number

Activity: **Ten Plus**

Tools: The Rekenrek

Directions: Knowing how to add 10 to a single digit number is an important number relationship strategy for addition. The Rekenrek is a visual tool for developing this strategy. With all of the beads on the student's right side, the teacher says, "Show me ten with one push." After the student pushes all ten from the top row to the left, the teacher says, "Now show me 12." After the student pushes two beads on the bottom row to the left, the teacher might reinforce the concept by saying, "10 and 2 more are 12." The teacher should continue posing 10+ problems until the student can answer them easily without the Rekenrek.

Activities to Support Problem Solving

The teacher can monitor a student's thinking about number by noting his/her solution strategies that are applied when solving various problem types. Solving problems provides a student with the opportunity to represent his/her understanding of the problem and to communicate his/her thinking. Having a student solve problems can help teachers understand the student's intuitive mathematical thinking and use that knowledge to help the student learn mathematics with understanding.

The teacher should refer to the student's responses on the Problem Type Interview to identify the student's current level of understanding about problem solving. By analyzing the student's responses, the teacher can understand more about:

- the student's ability to understand the language of the problem type,
- the number sizes that are most meaningful to the student,
- the strategy the student uses and,
- the student's reliance on the context both in terms of the story and the counters used to represent the story.

The teacher needs to continuously modify instruction in the language, number sizes, and context of the problem to account for the small steps a student makes as she/he develops his/her mathematical thinking.

Problem solving begins with sharing, counting, and organizing situations that are part of a student's daily experience. When a student counts out snacks, organizes for games, and thinks about home/school lunch counts, he/she is problem solving. While a student usually has good ideas regarding what is involved in these concepts, he/she may not have connected the experience with mathematics. The teacher can help a student make these connections.

As the student matures, the teacher can substitute counters for the objects in the problem and can introduce contexts that are not as immediate or naturally occurring. (For example: there are 2 trees and each tree has 5 apples, how many apples are there?)

A student's first problem solving strategy is direct modeling, a counting all strategy. Instruction in story problem solving for the student receiving intervention in first grade should focus on the following problem types: join result unknown, separate result unknown, multiplication, and measurement division.

Students are beginning to make a shift in their thinking when they count forward and backward from any number and name up to three numbers after and before a given number (within the Level B or Level C range). As teachers observe these changes they can extend problem solving experiences with number ranges and problem types that encourage the development of the counting on and counting back from strategies.

Having students continue to engage in story problem solving as they begin to develop base ten understanding is critical. The compare difference unknown, the join change unknown and the part-part whole problem types when used with decade and single digit number combinations, assist the student in forming base ten understandings. Additionally, teachers can use multiplication story problems or partitive and measurement division story problems that involve groups of ten to further refine this base ten understanding. Teachers should re-administer the Problem Type Interview periodically to guide the introduction of other problem types and number sizes.



Number Development Assessment

Name: _____

School: _____ Teacher: _____

Date of Interview: _____ Interviewer: _____

1. Forward Number Sequence (Say: "Start counting from _____and I'll tell you when to stop.")

Level 1: (a) 1 (to 32) _____ (b) 8 (to 17) _____

(c) 22 (to 30) _____

Level 2: (a) 47 (to 53) _____ (b) 77 (to 83) _____

Level 3: (a) 96 (to 112) _____

2. Number After (Say: "Tell me the number that comes right after _____. For instance, if I say 1, you would say _____?")

Level 1:	2	5	9	12	19		
Level 2:	49	29	50	80	59	79	39
Level 3:	109						

3. Backward Number Sequence (Say: "Count backwards, like 3, 2, 1.")

Level 1: (a) 10 (down to 1) _____ (b) 15 (down to 10) _____

Level 2: (a) 22 (down to 16) _____ (b) 33 (down to 26) _____

(c) 62 (down to 56) _____ (d) 85 (down to 77) _____

Level 3: (a) 112 (down to 99) _____

4. Number Before (Say: "Tell me the number that comes just before _____.For instance, if I say 2, you would say _____?")

Level 1:	3	5	9	14	20
Level 2:	41	89	60	69	100
Level 3		110			

5. Numeral Identification (Say: "What number is this?)

Level 1:	8	3	5	7	9	2	4	6	1	10
Level 2:	24		29							
Level 3:	12		20	83		14	81	13	21	15
Level 4:	340		213	850		620	380			

6. Sequencing Numbers (Say: "Please put these numbers in order from smallest to largest, starting here." – point to left of workspace. Ask child to identify the numerals in order after sequencing)

Level 1: 1-10 _____ All correctly identified?

Level 2: 8-17 _____ All correctly identified?

Level 3: Decade cards for 10-100 _____ All correctly identified?

Level 4: 64-73 _____ All correctly identified?

7. Subitizing (Say: "I'm going to show you some dots really quickly. I want you to tell me how many there are." Show each dot pattern 1-2 seconds.)

Level 1: (regular dot patterns)	2	4	3	5	6
Level 2: (irregular dot patterns)	3	4	5	6	
Level 3: (regular dot patterns)	7	8			
(irregular dot patterns)	7	8			

8. Additive Tasks Note: Use one color counter for the covered set and a different color for the added set.

Level 1, part 1: Count out a set of 18 counters (same color)_____

Level 1, part 2: Say: "I have _____ counters under here (show the child the counters and then cover) I am going to slide one more under (the child should watch the counter being slid under), how many are under here now?"*

(a) 3 and then slide one more _____ (b) 7 and then slide one more _____

(c) 11 and then slide one more _____

Level 2: Say: "I have _____ counters under here and _____ more counters over here" (leave counters exposed adjacent to cover). Wave your hand over both sets. "How many altogether?" *

(a) 3 covered, 1 exposed _____

(b) 4 covered, 2 exposed _____

(c) 5 covered, 4 exposed _____

(d) 12 covered, 3 exposed _____

(e) numeral 22 covered, 2 exposed _____

Level 3: Say: "I have _____ counters under here (show and then cover set) and I have _____ counters under here. (show and then cover set). How many altogether? (Wave your hand over both covered sets) *

(a) 5 + 2 _____

(b) 7 + 5 _____

(c) 15 + 3 _____

(d) numeral 25 + 3 (counters) _____

9. Missing Addend Tasks (Say: "Here are _____ counters. (Cover) Look away. I am putting some more under here. Now there are _____. How many more did I put under here?)
Note: Use two colored counters with the original set in one color and the added items in another

(a) 4 to 5 (secretly put 1 under) _____

(b) 5 to 7 (secretly put 2 under) _____

(c) 6 to 9 (secretly put 3 under) _____

(d) 15 to 17 (secretly put 2 under) _____

10. Tens and Ones Tasks (Depending on the child's familiarity with the tool, you may cubes organized in ten sticks, ten frames, base ten blocks or a picture of these to assess this understanding.)

Part one: Place 4 unit cubes in front of student. Ask "How many?" Then place a group of ten, base ten blocks, or a ten frame, next to the unit cubes, ask "Now, how many?" Continue placing tens until you reach 74. Circle the correct answers given.

4 14 24 34 44 54 64 74

Part Two: Place the following sequence of cubes, base ten blocks, ten frames, or a representation of them, in a line in front of child: 10, 3, 10, 10, 4, 3, 10, 2, 10, 10. Cover everything. Slowly uncover each set and ask the child to add that next quantity to the total. Circle the correct answers given:

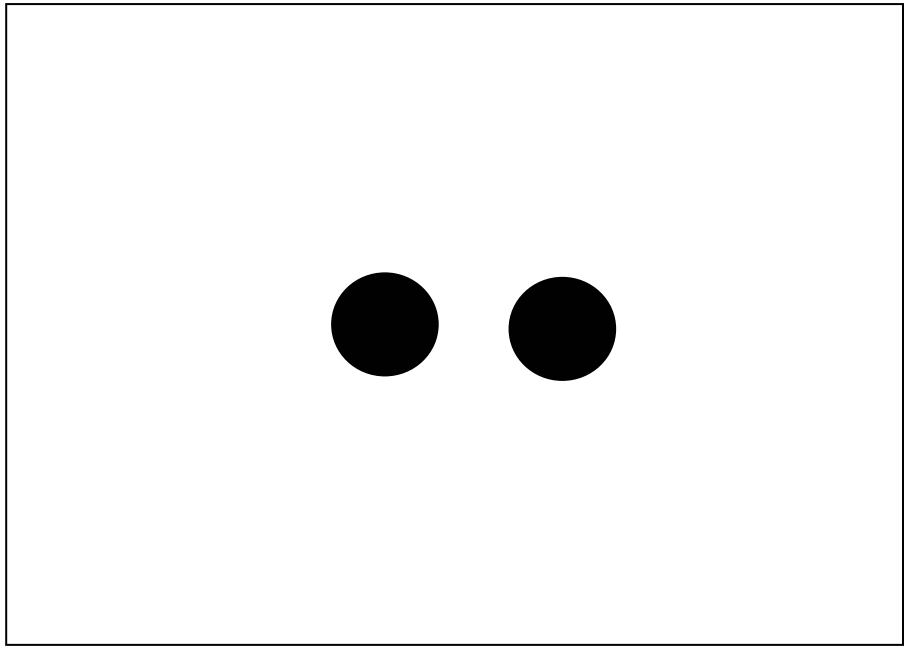
10 13 33 37 40 50 52 72

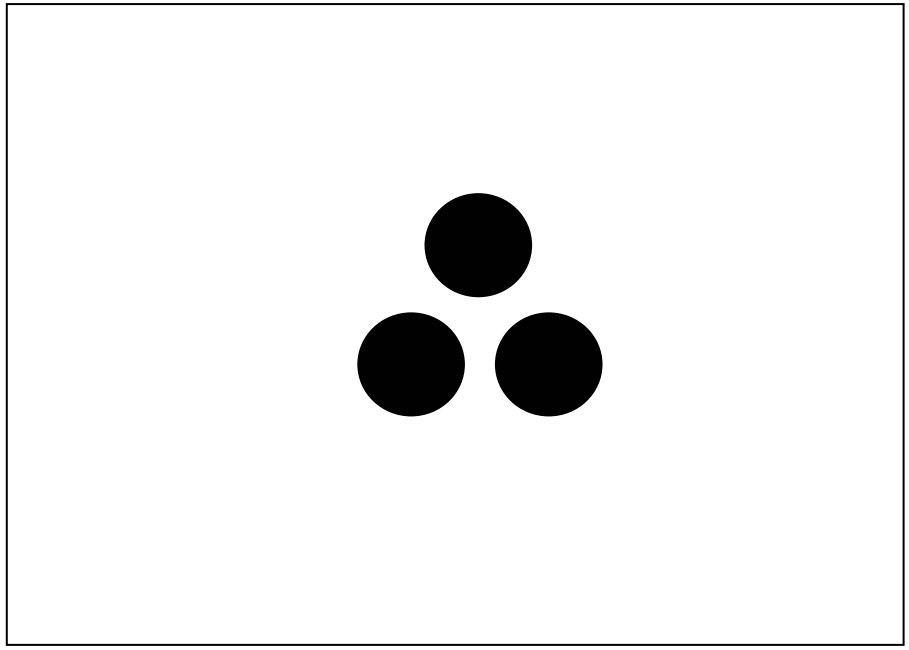
Part Three: Place the following sequence of cubes, base ten blocks, ten frames, or a representation of them, in a line in front of the child: 4, 10, 20 , 12 , 25 Cover everything. Slowly uncover each set and ask the child to add that next quantity onto the total. Circle the correct answers given below:

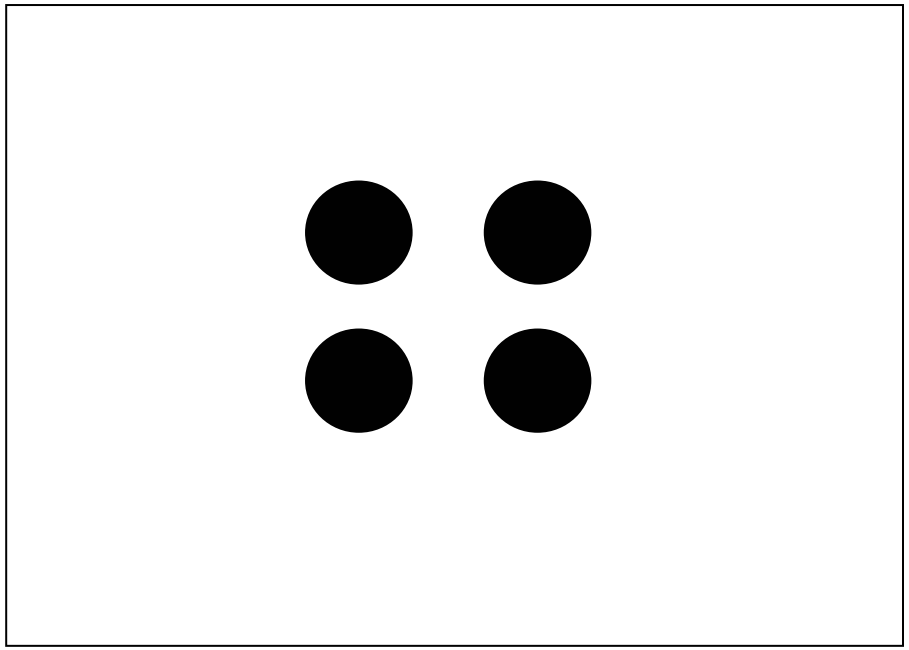
4 14 34 46 71

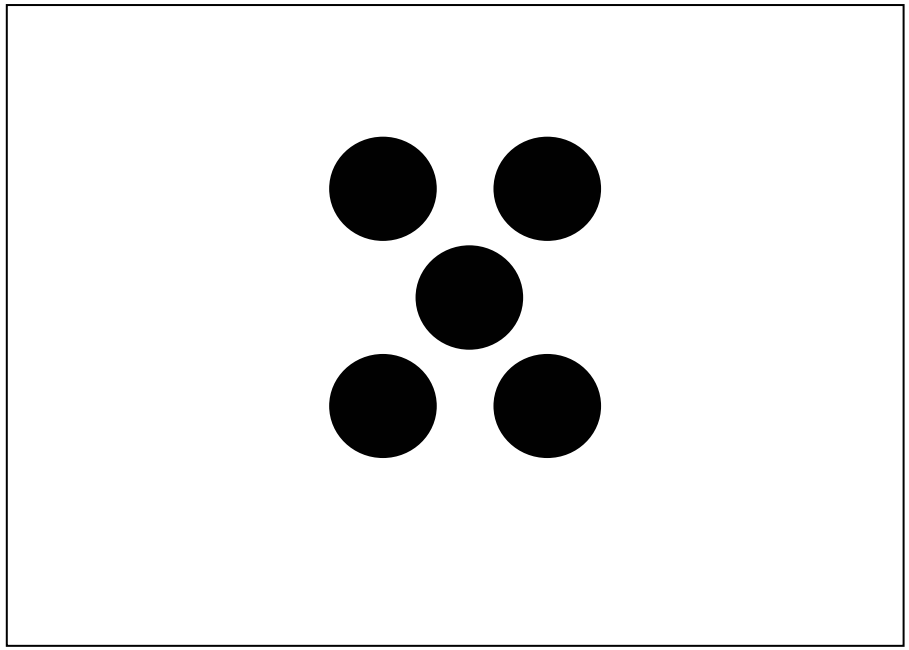
Notes:

Dot Cards (p. 1 of 4)

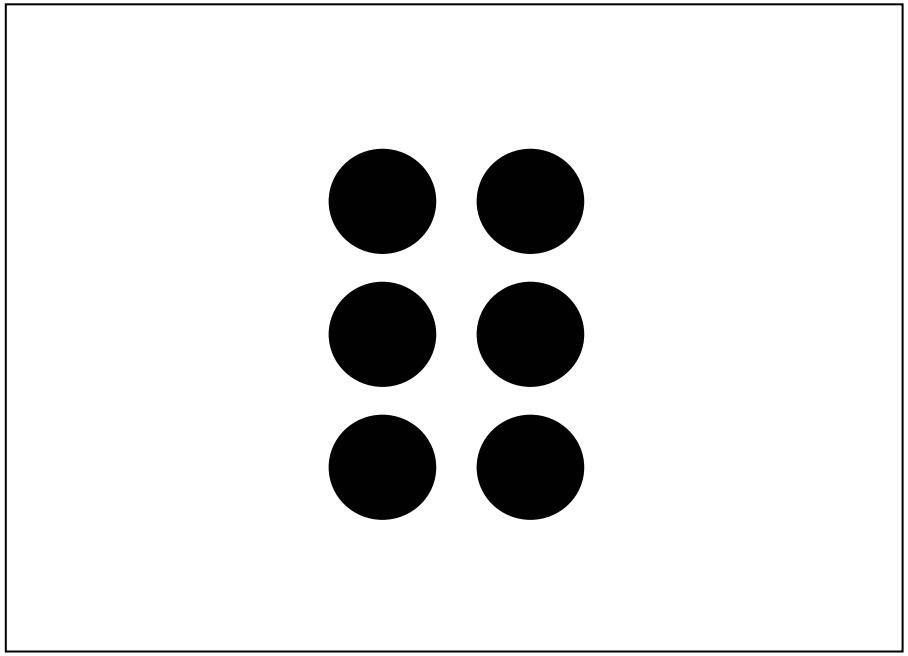


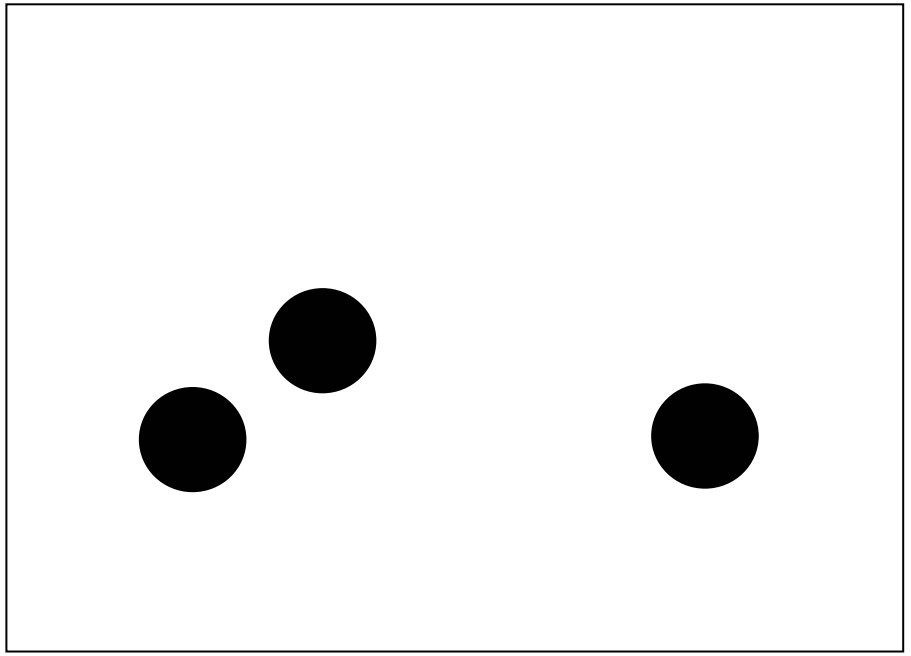


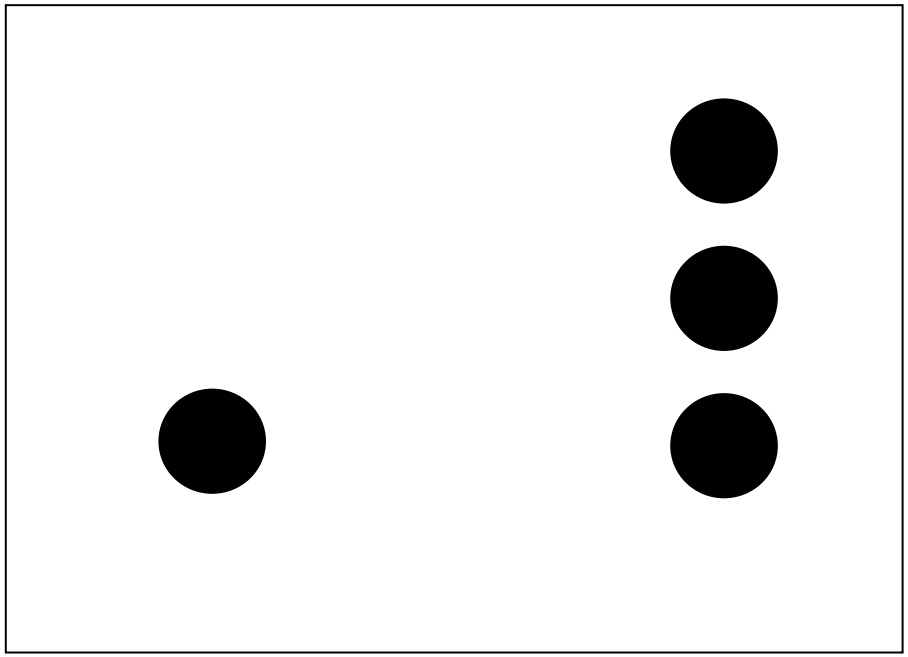


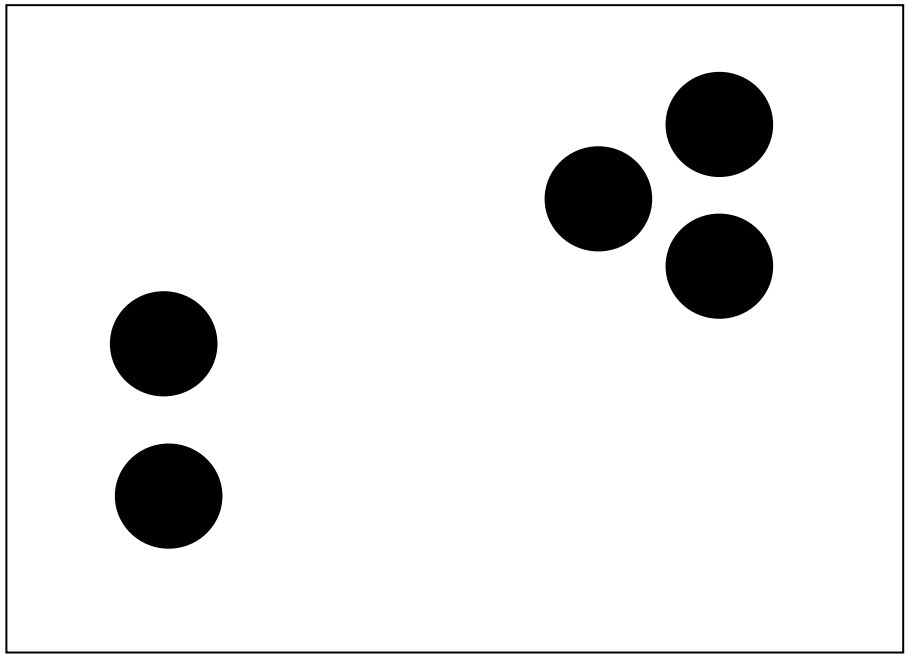


Dot Cards (p. 2 of 4)

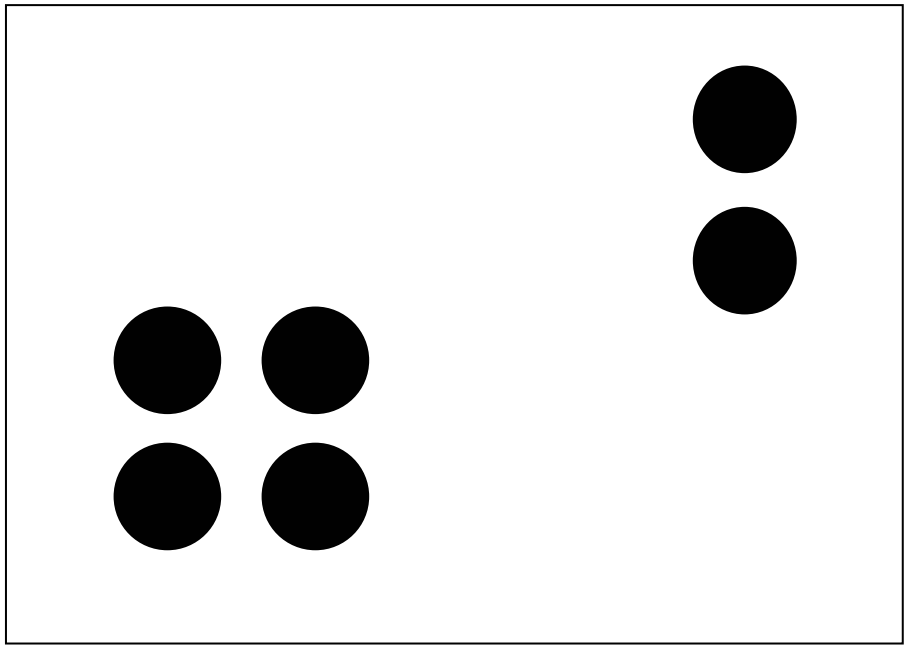


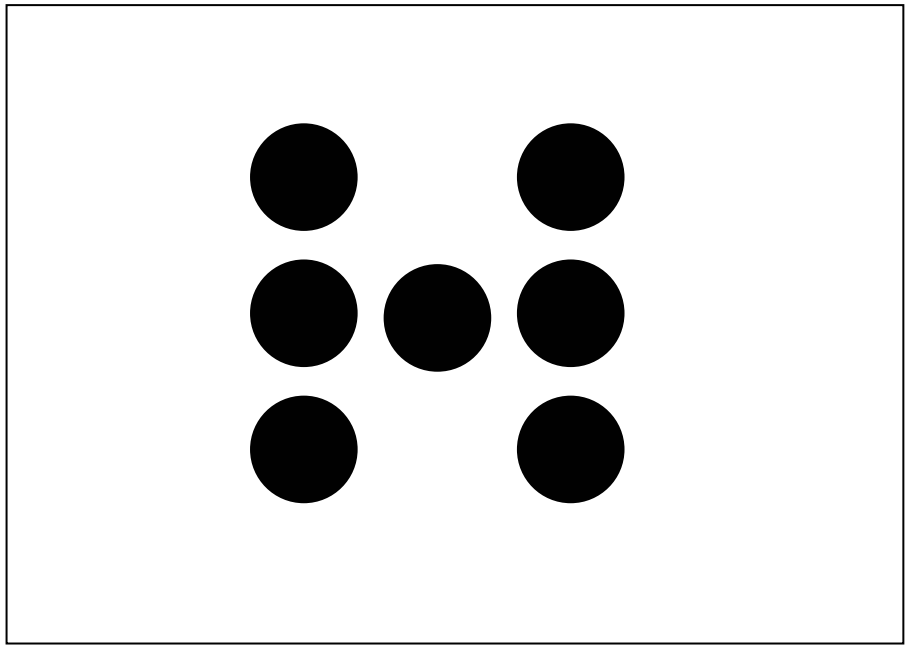


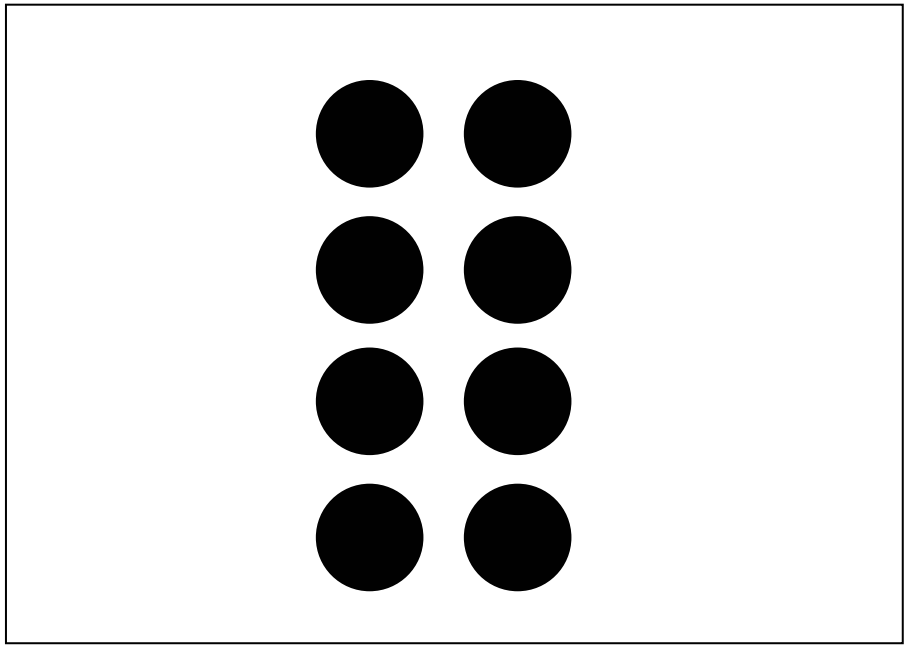


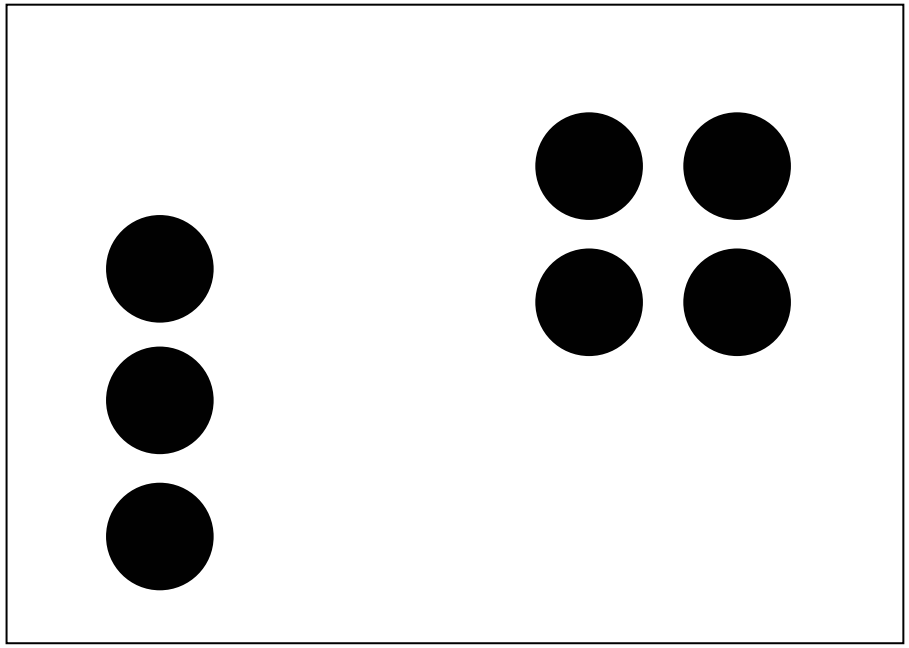


Dot Cards (p. 3 of 4)

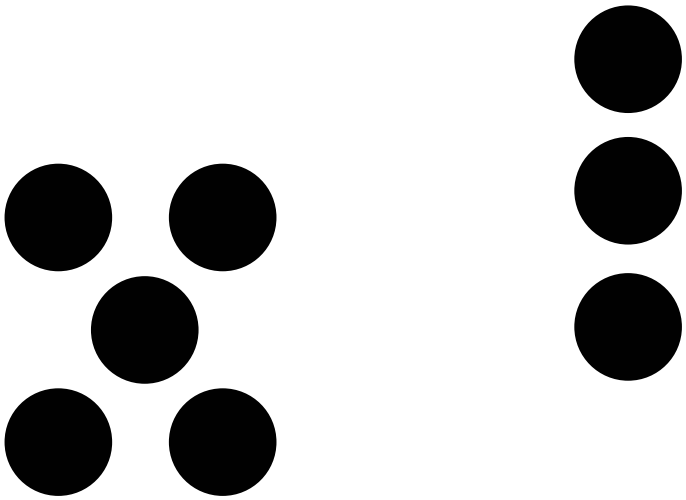








Dot Cards (p. 4 of 4)



Number Development Assessment—Number Identification

8 3 5 7 9 2 4 6 1 10

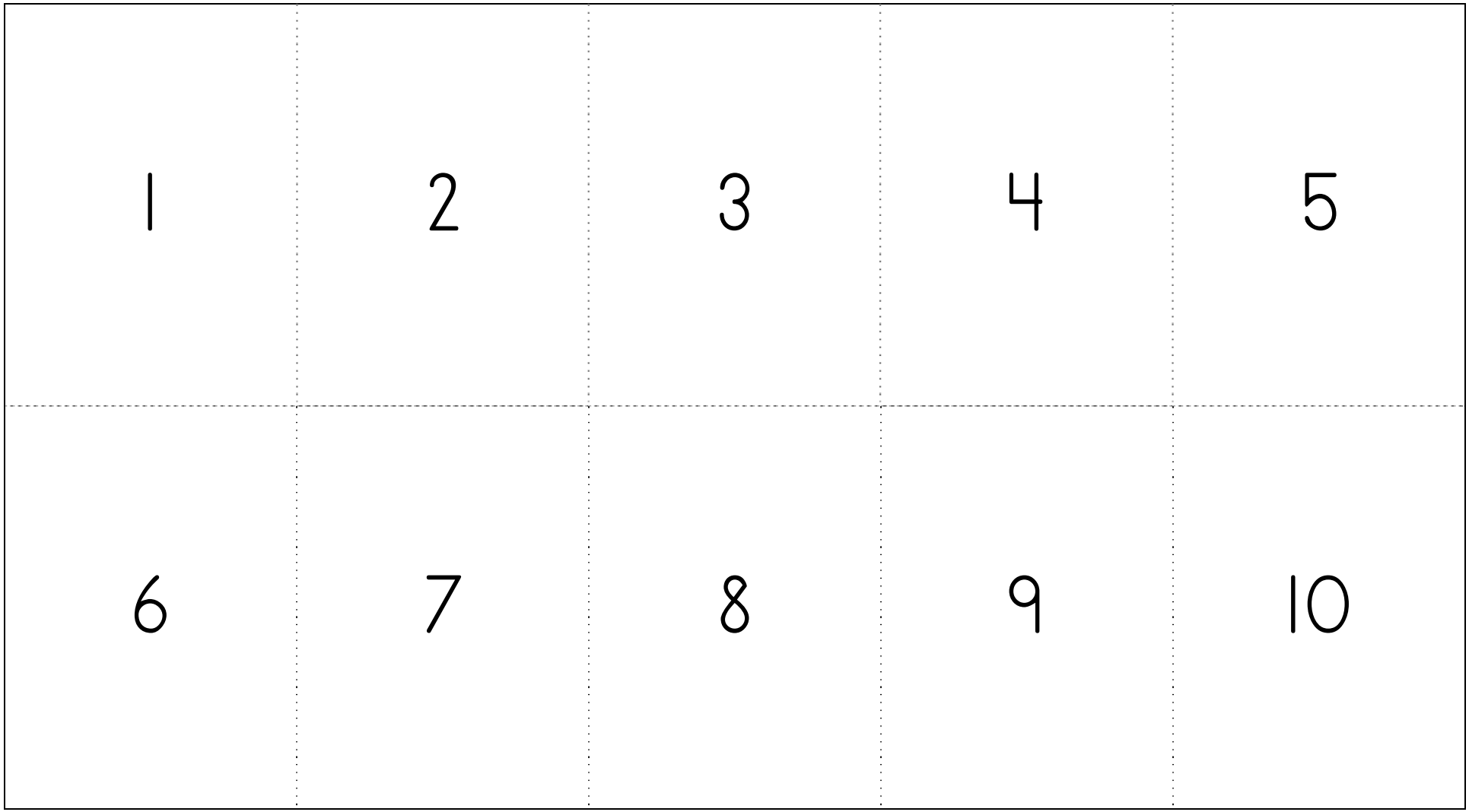
24 29

12 20 83 14 81 13 21 15

340 213 850 620 380

Number Development Assessment

Sequencing Level 1



Number Development Assessment

Sequencing Level 2

8	9	10	11	12
13	14	15	16	17

Number Development Assessment

Sequencing Level 3

10	20	30	40	50
60	70	80	90	100

Number Development Assessment

Sequencing Level 4

64	65	66	67	68
69	70	71	72	73

Planning & Progress Record

Student: _____

Concept/skill	**Date** *Activity and number range*	**Progress** **E**: *emerging* **P**: *progressing* **I**: *inconsistent* **C:** *consistent*	**Comments**
Forward counting *(including number after)*			
Backward counting *(including number before)*			
Sequencing numbers			
Number identification			

Planning & Progress Record

Student: _____

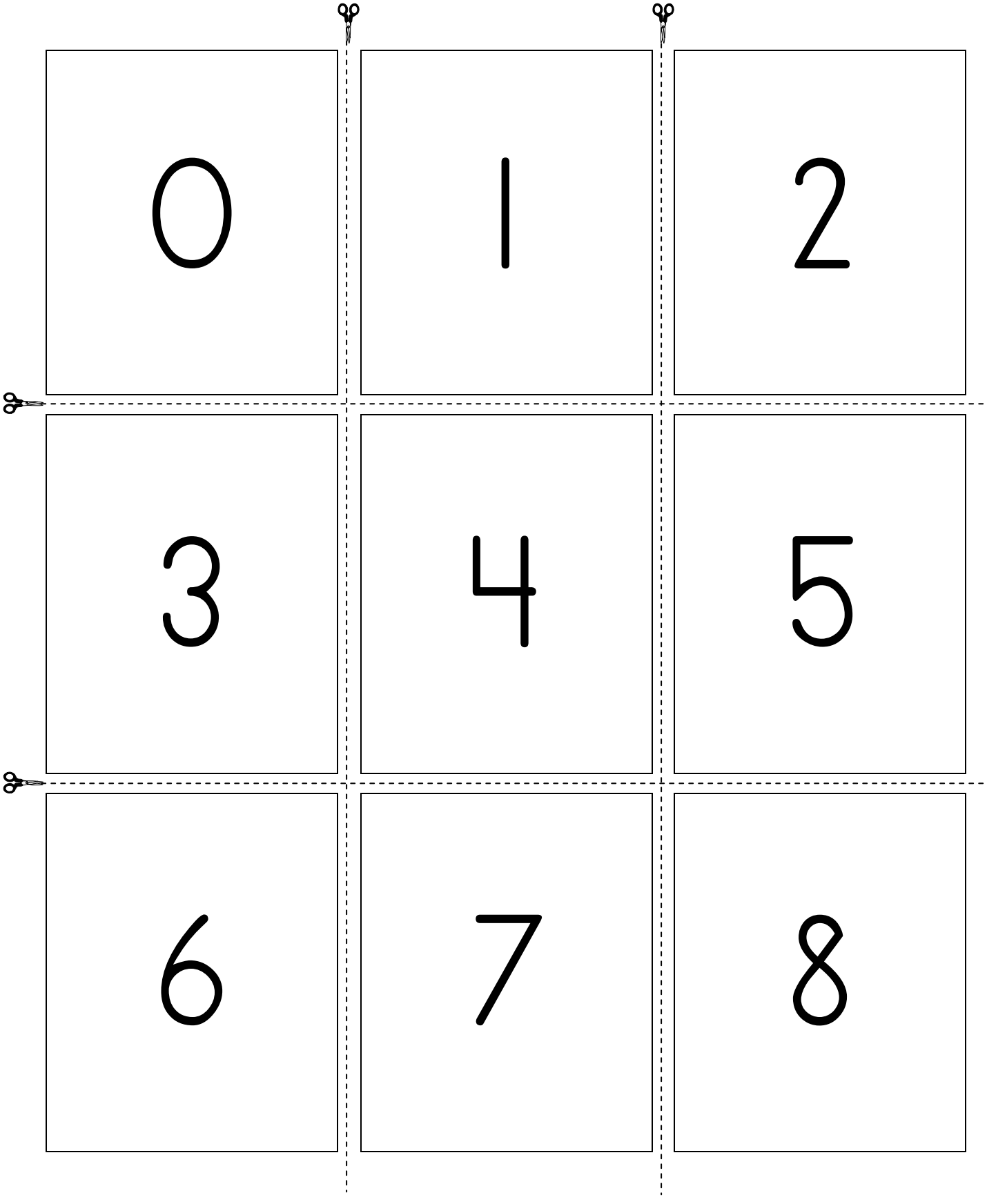
Concept/skill	**Date** *Activity and number range*	**Progress** **E**: *emerging* **P**: *progressing* **I**: *inconsistent* **C:** *consistent*	**Comments**
Composing/ Decomposing Numbers			
Counting On & Counting Back strategies			
CGI story problems *(Indicate problem type & number range)*			

Teacher: _____ Child: _____

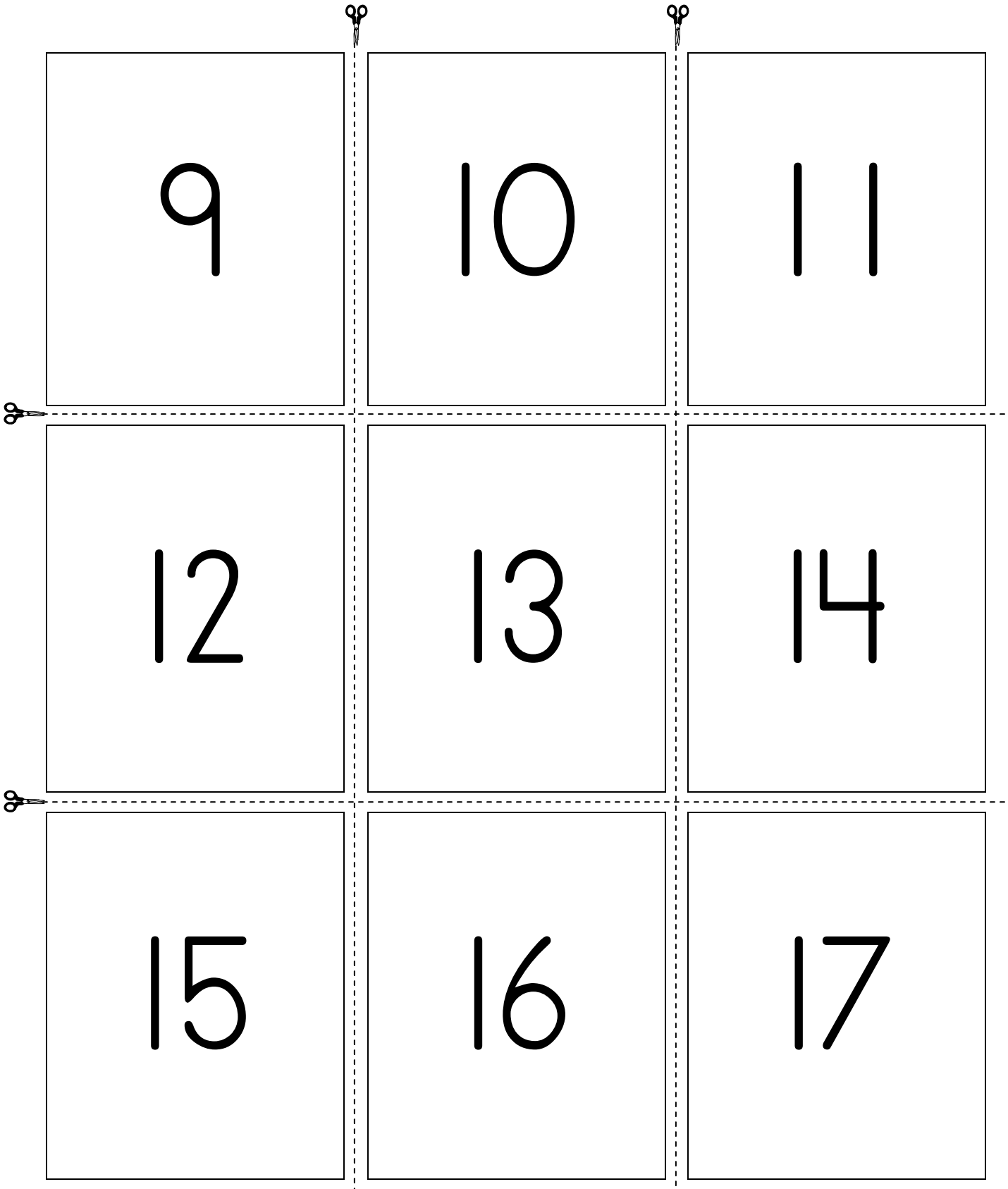
Log of Intervention Tried

Date	Intervention Activity	Progress of child

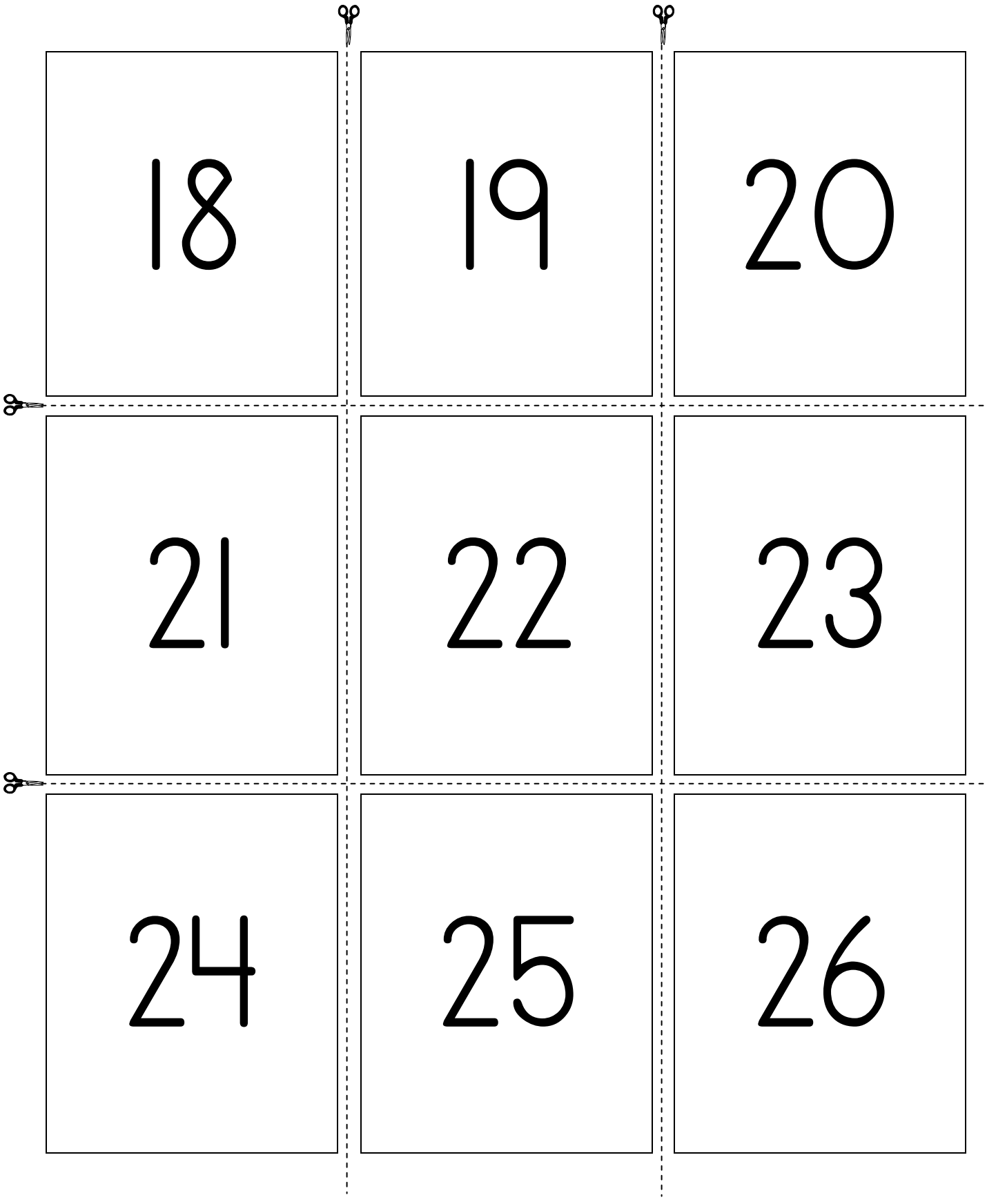
Number Cards 0—8



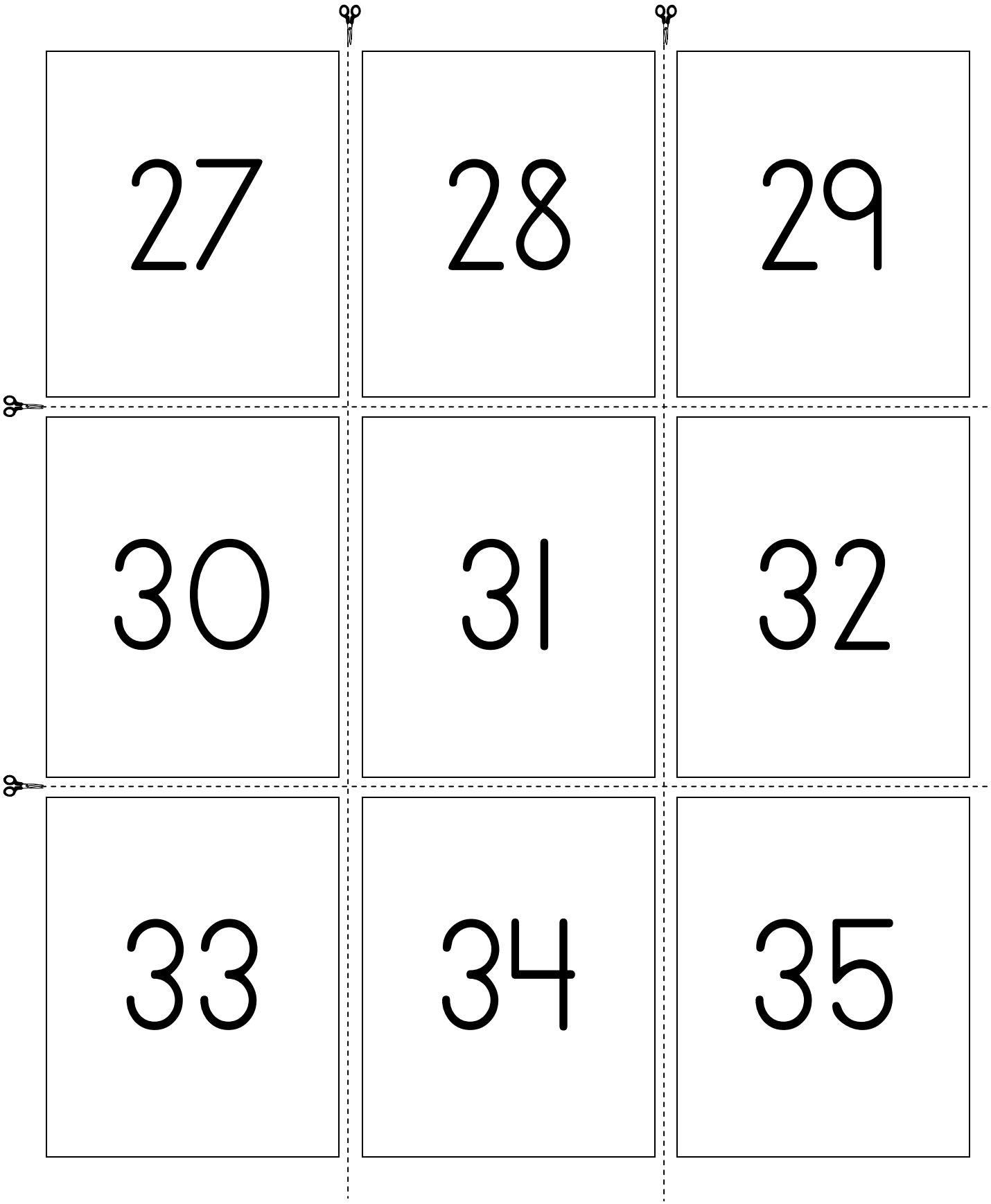
Number Cards 9—17



Number Cards 18—26



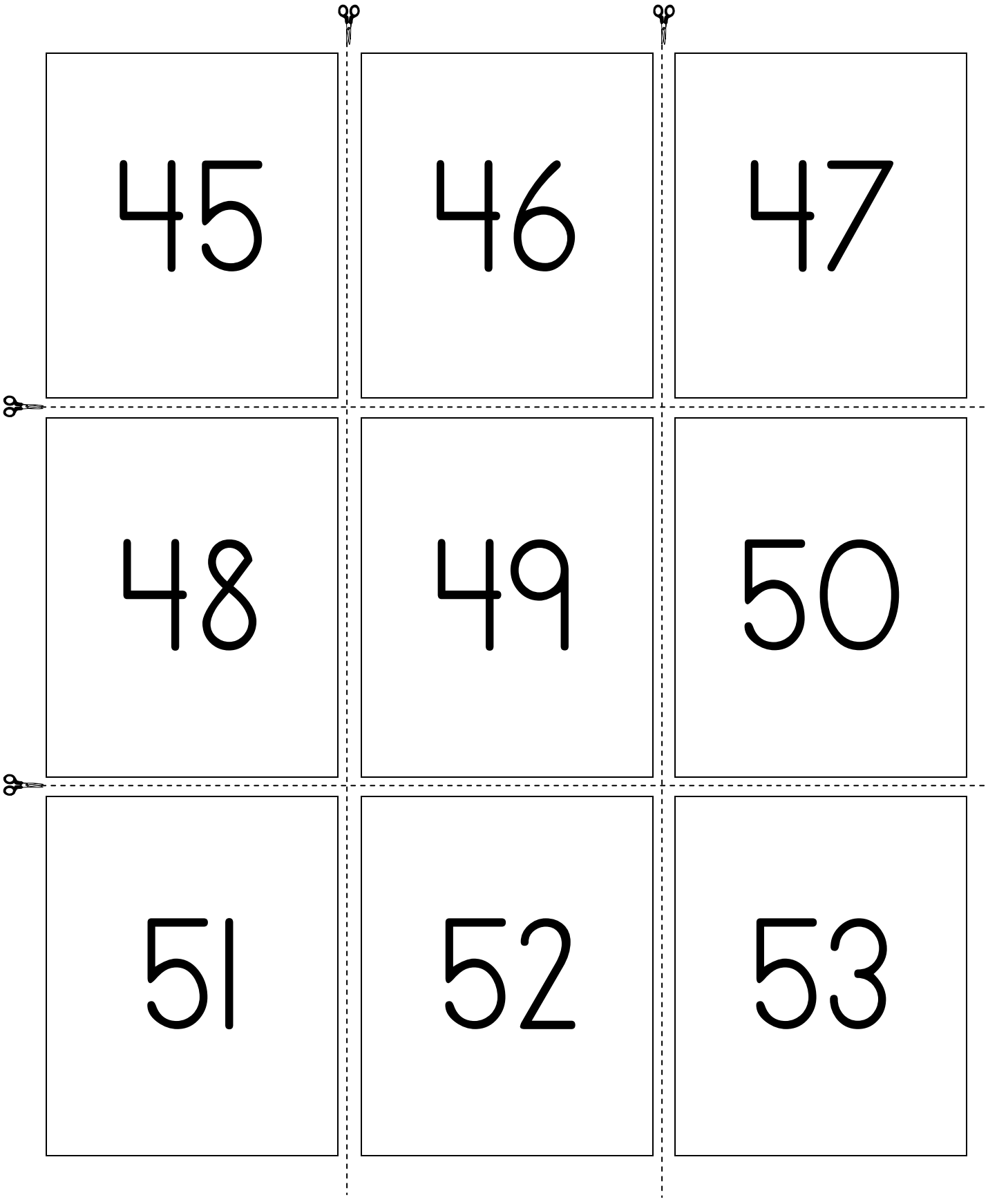
Number Cards 27—35



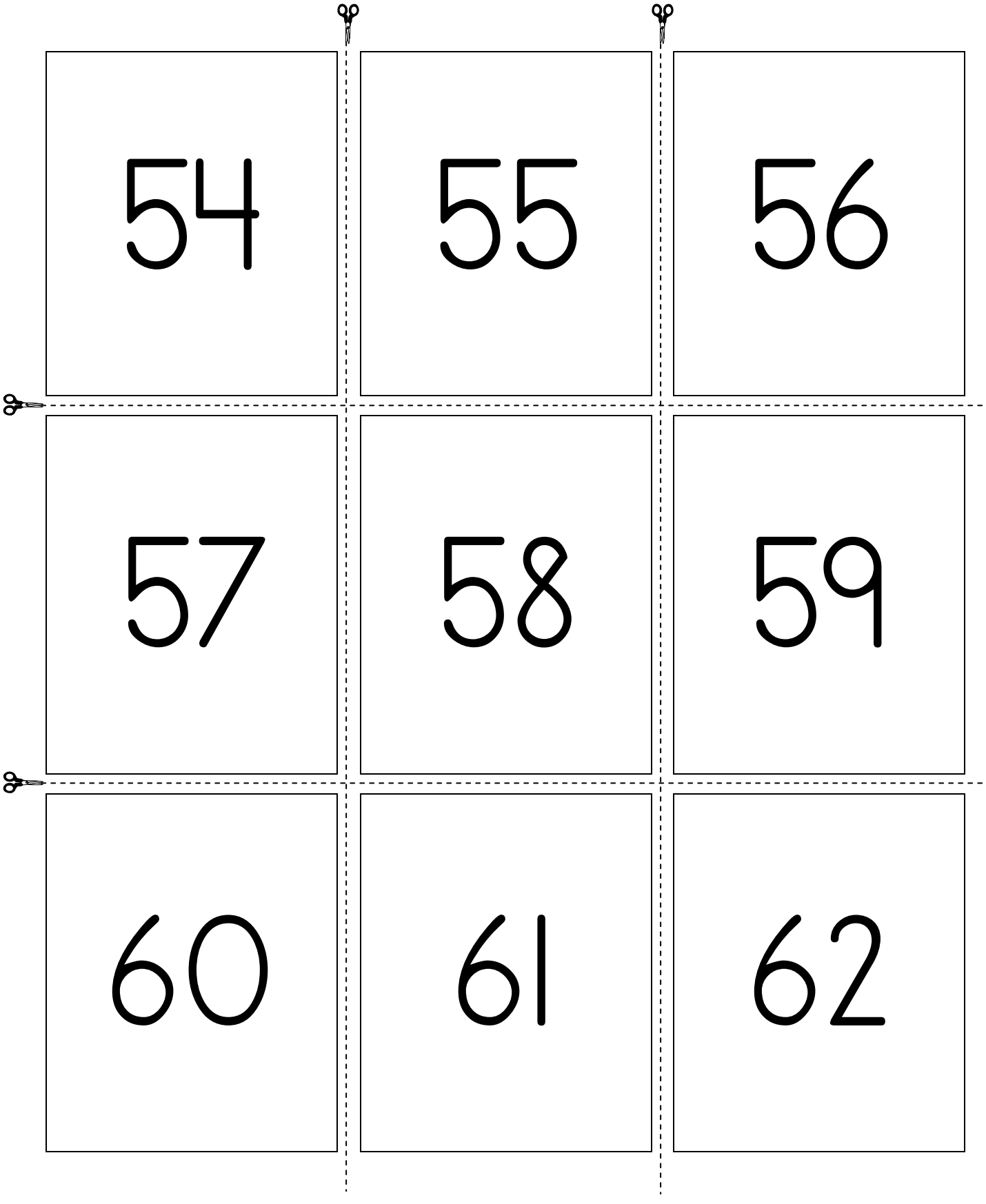
Number Cards 36—44

36	37	38
39	40	41
42	43	44

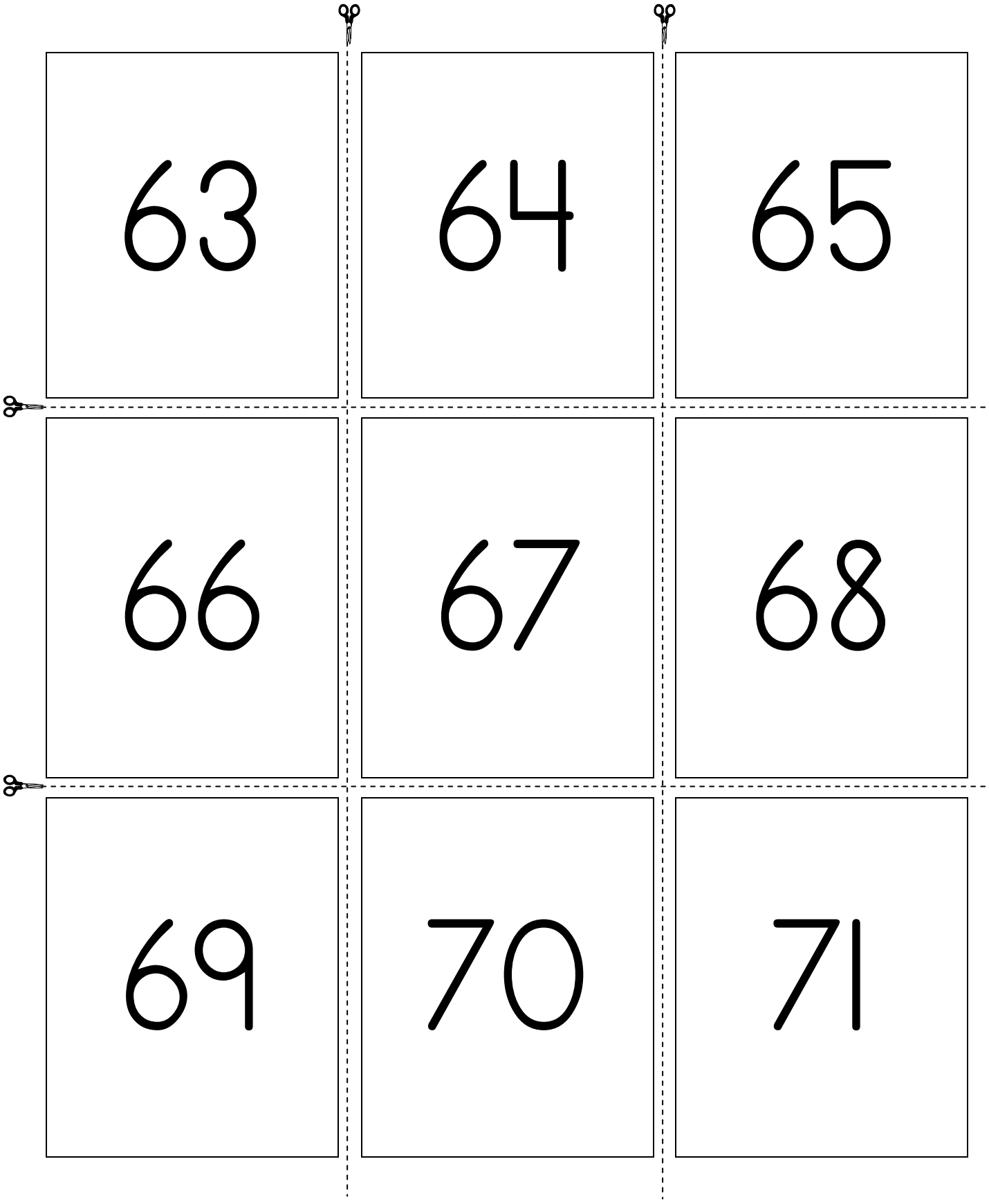
Number Cards 45—53



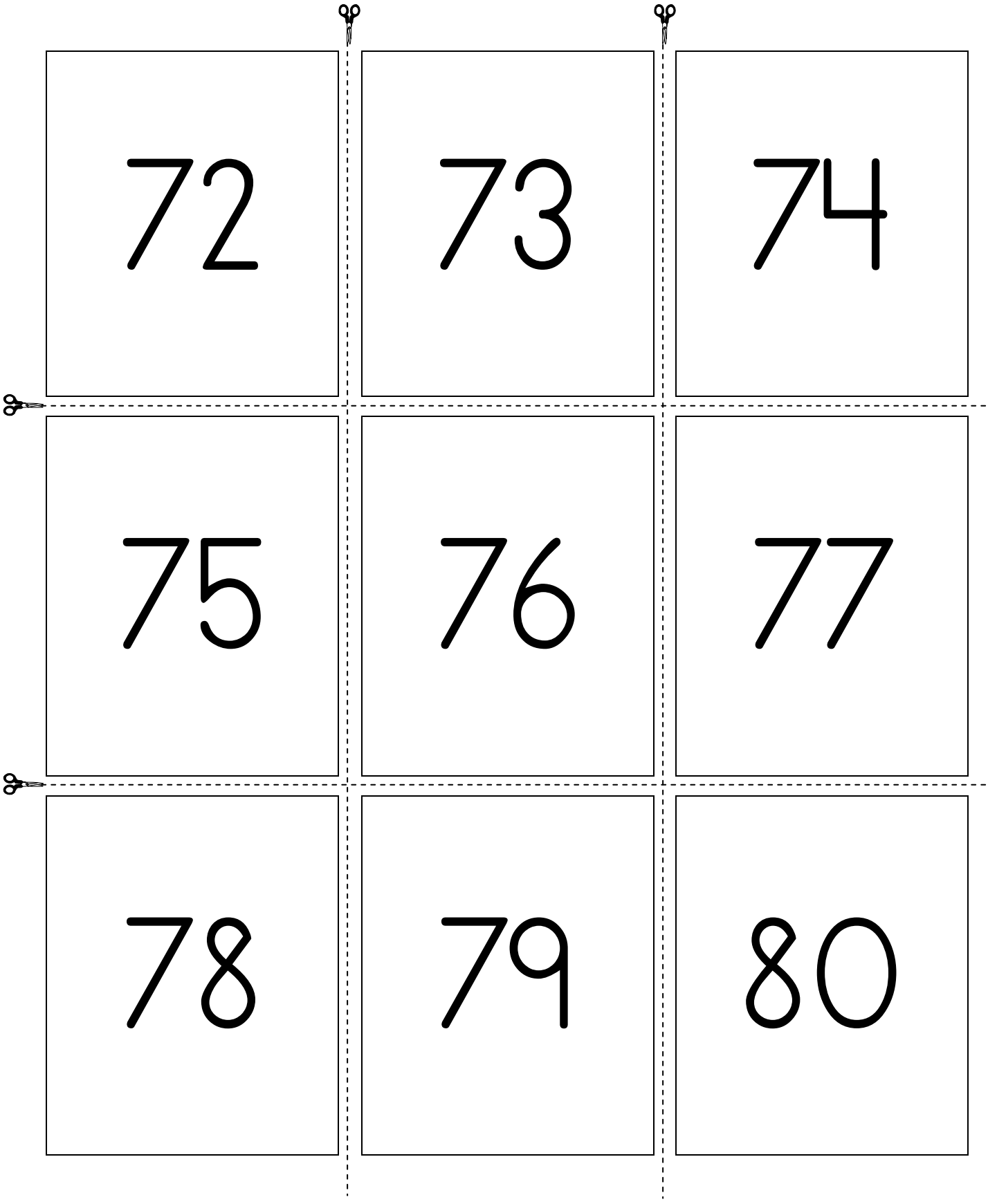
Number Cards 54—62



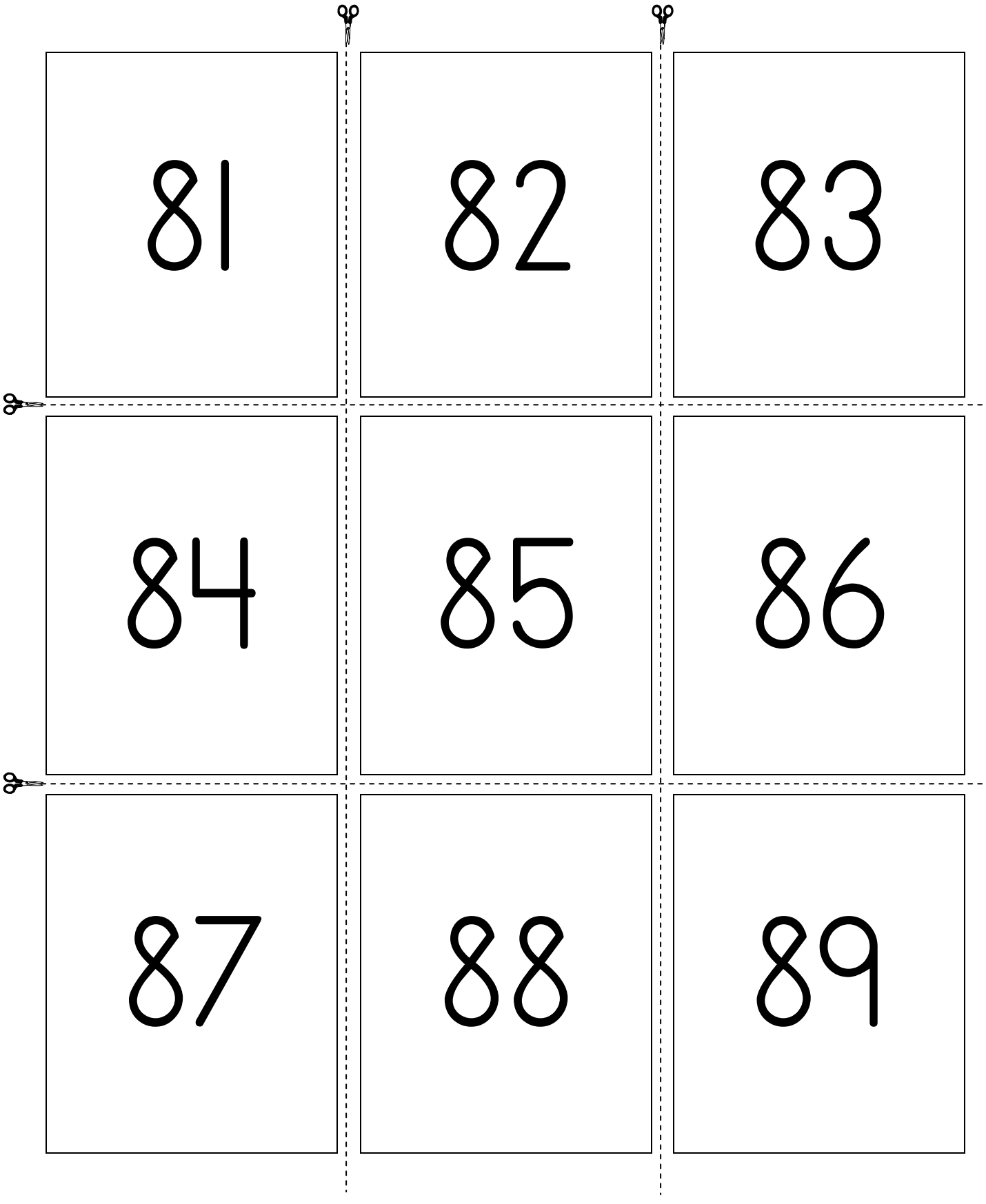
Number Cards 63—71



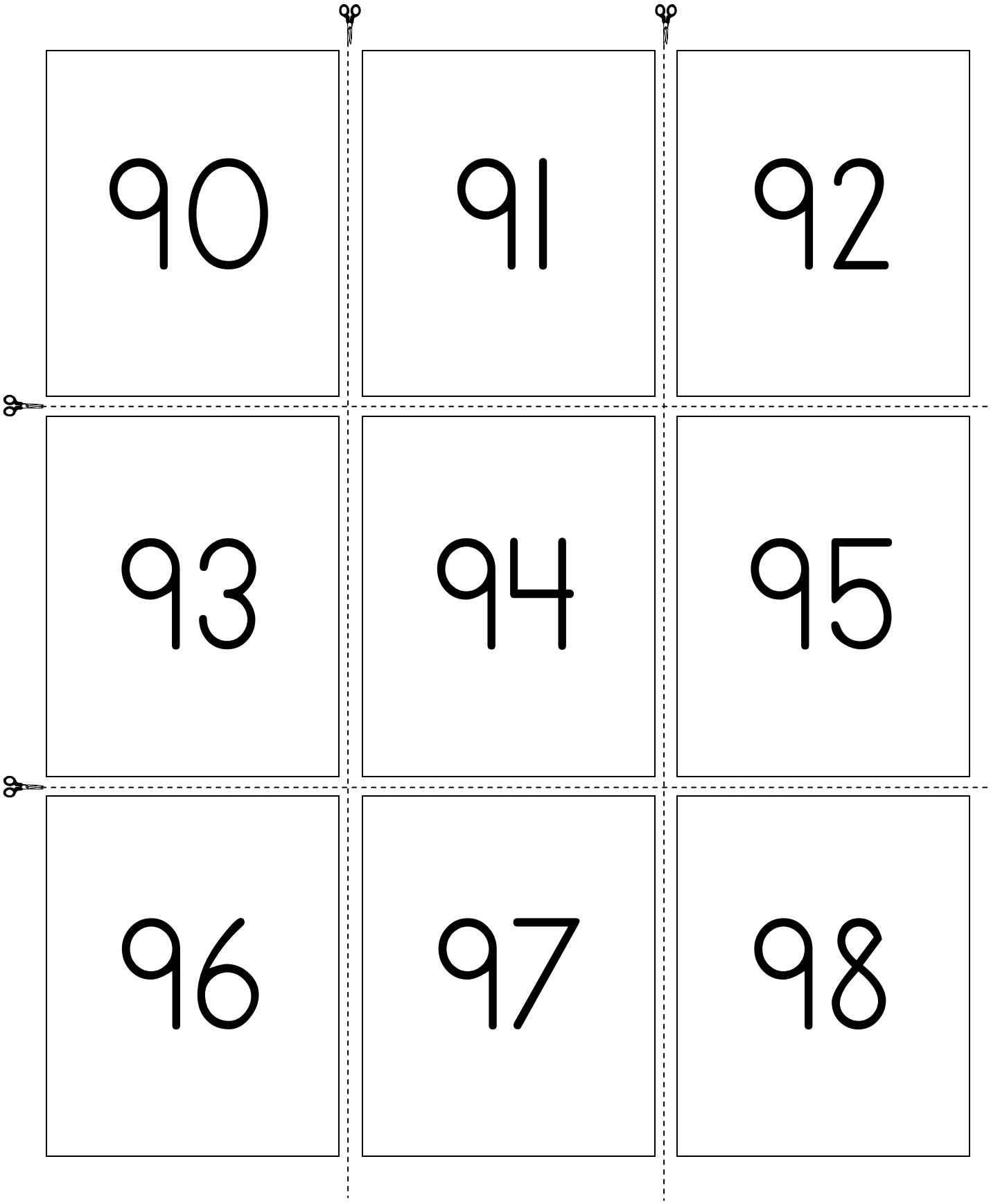
Number Cards 72—80



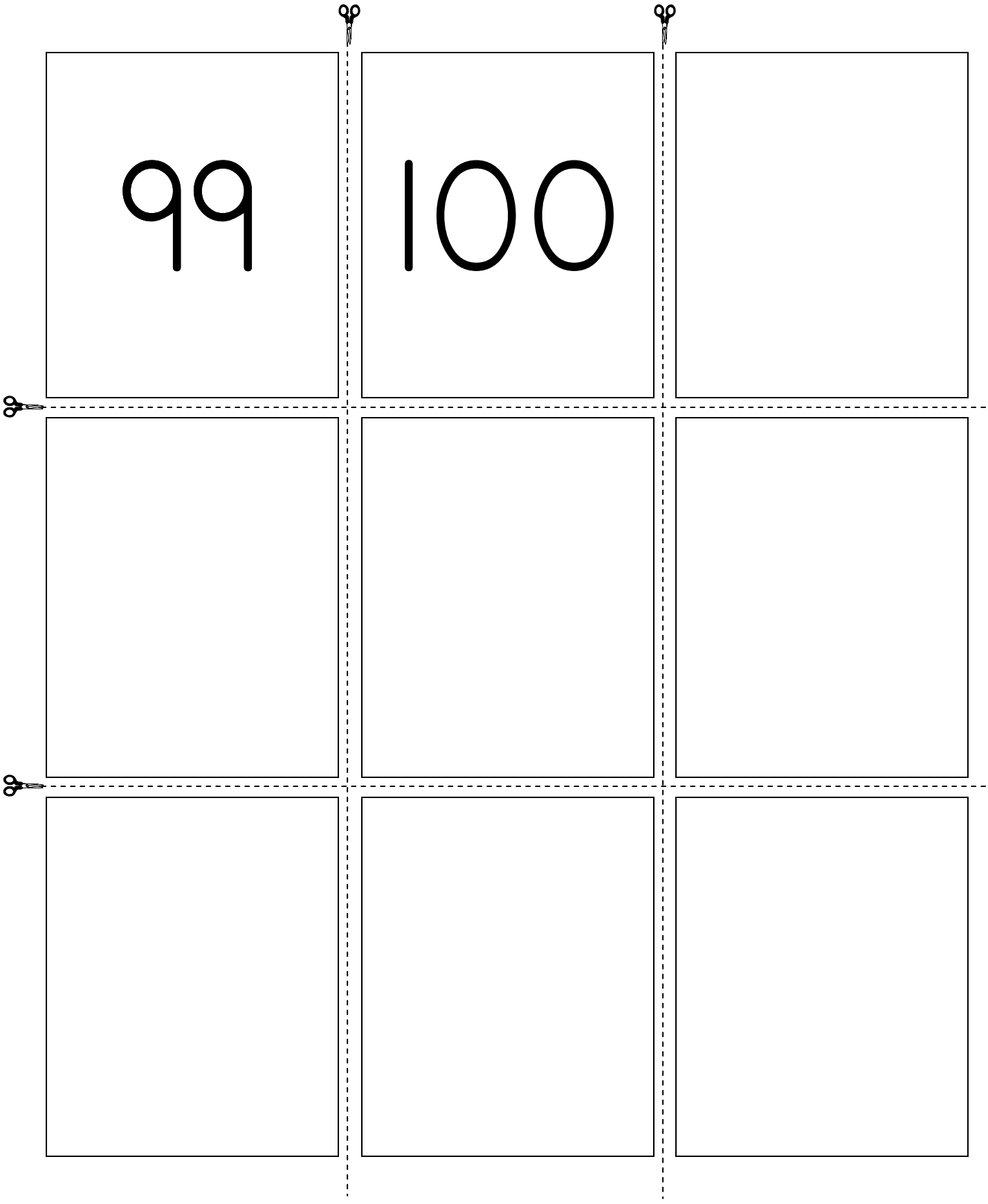
Number Cards 81—89



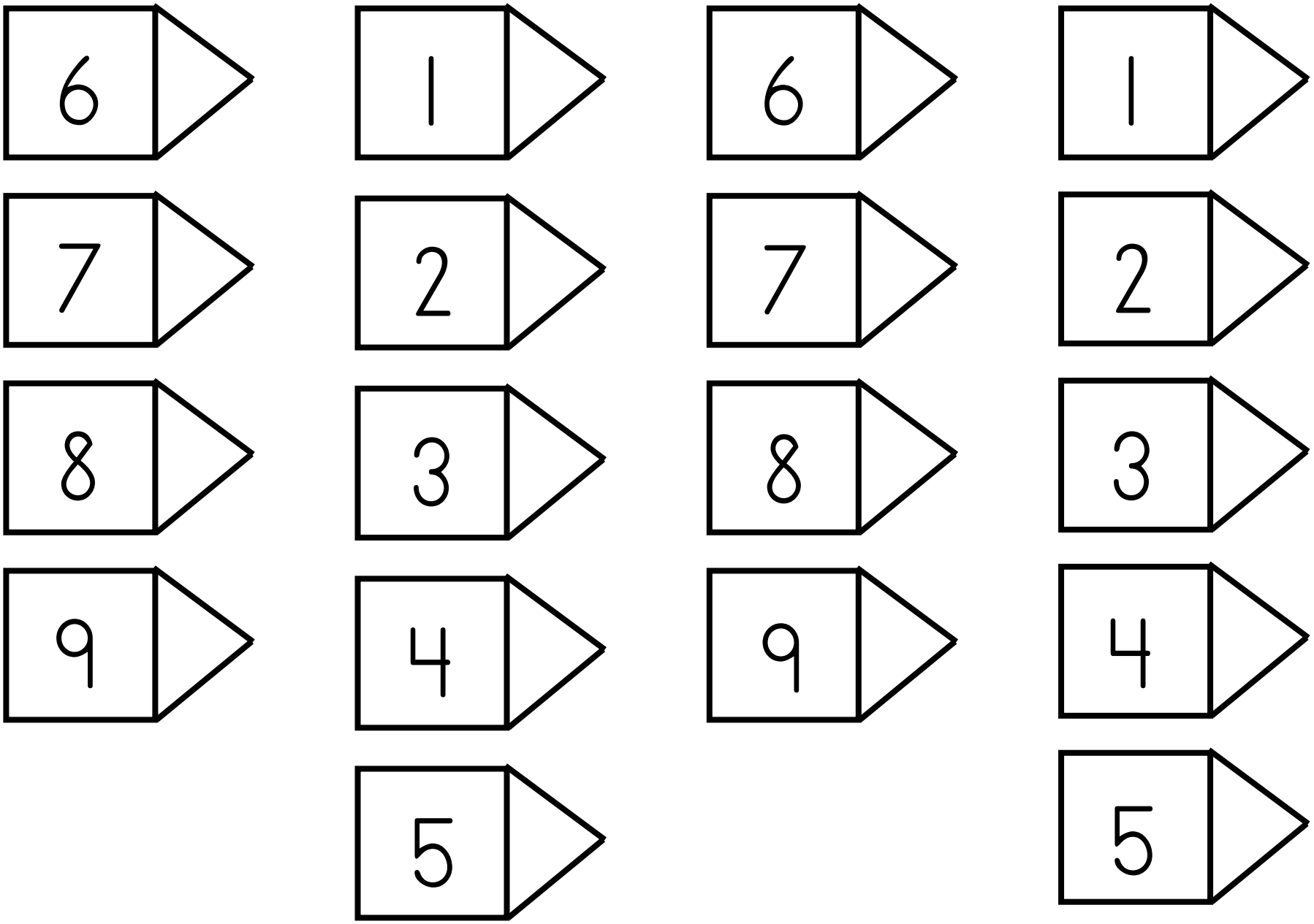
Number Cards 90—98



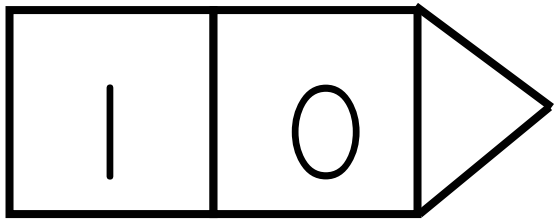
Number Cards 99—100

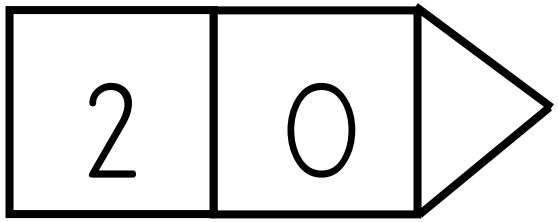


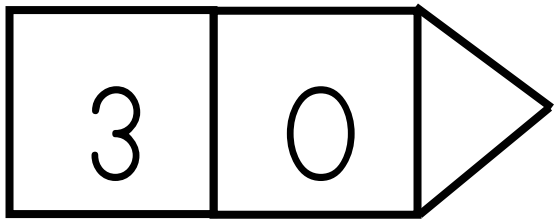
Arrow Card Directions (page 1): Copy onto colored paper. Cut out a set for each child.

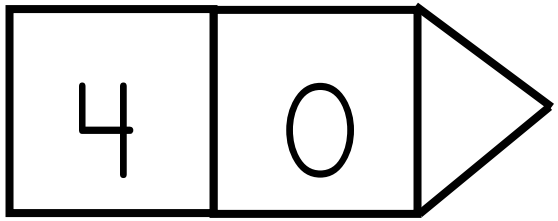


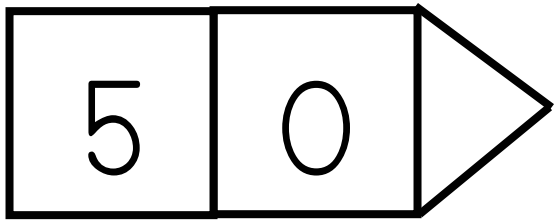
Arrow Card Directions (page 2): Copy onto colored paper. Cut out a set for each child.

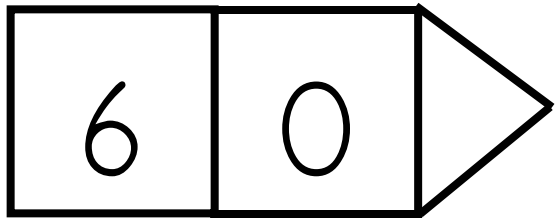


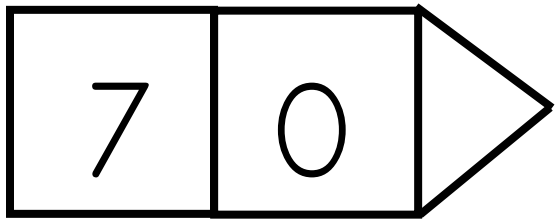


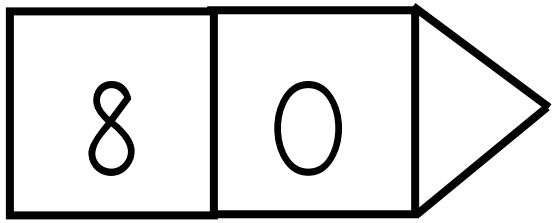


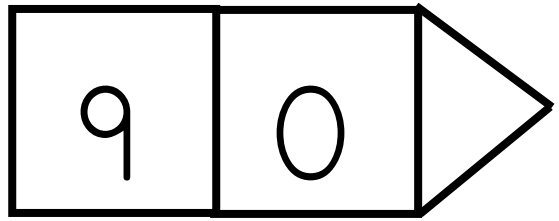




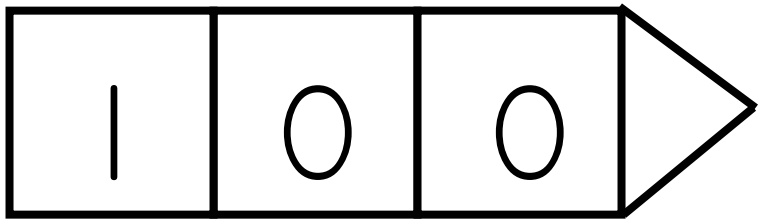


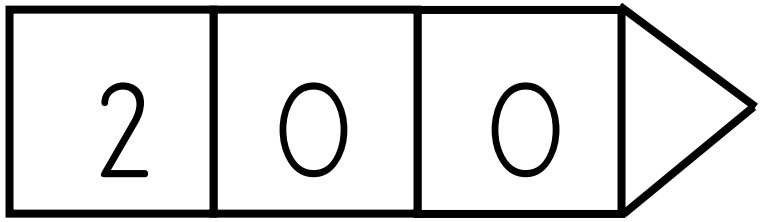


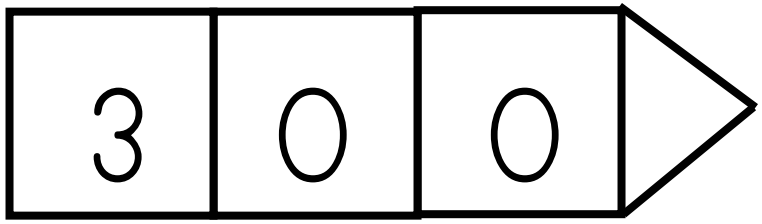


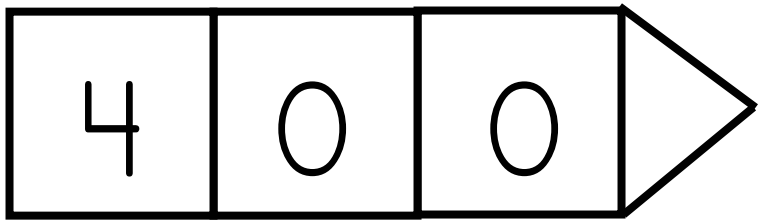


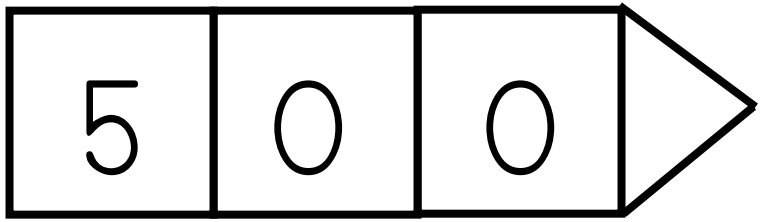
Arrow Card Directions (page 3): Copy onto colored paper. Cut out a set for each child.

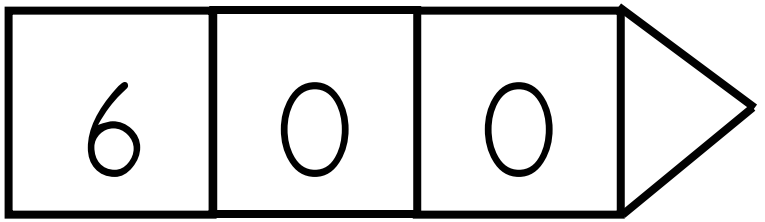


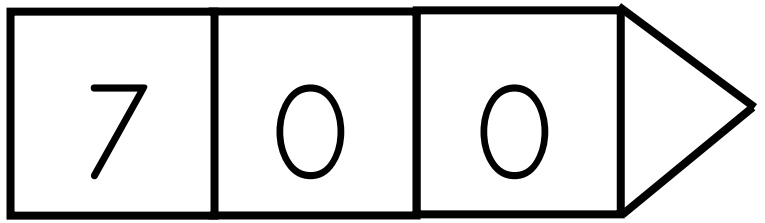


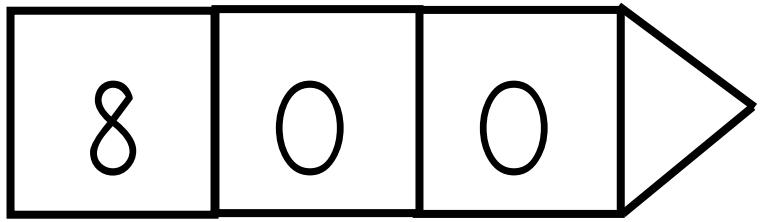


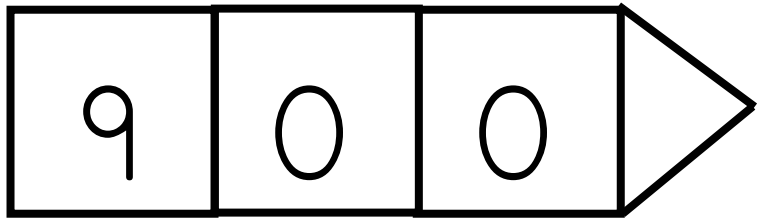




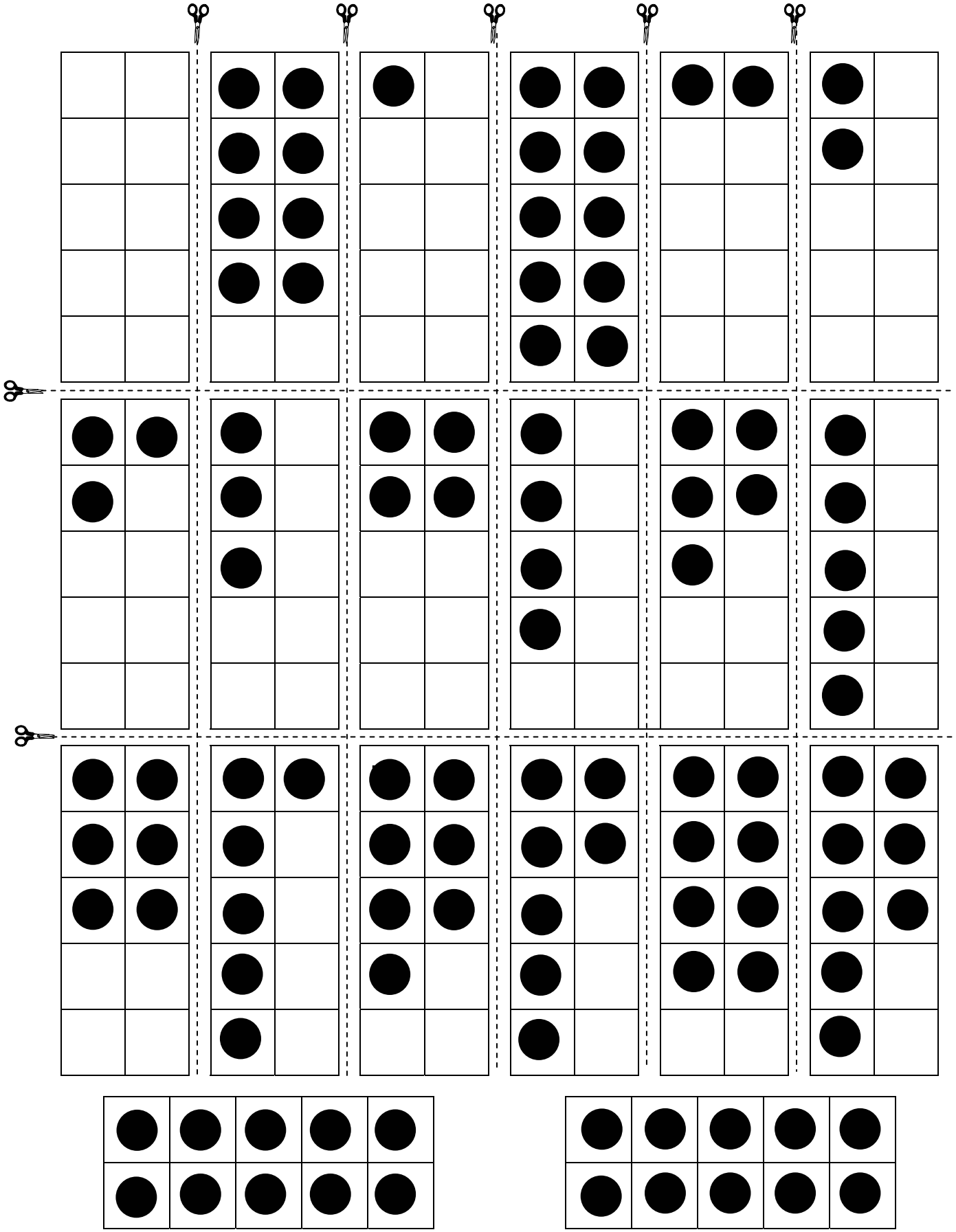




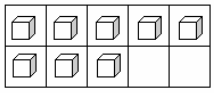


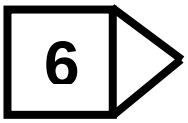


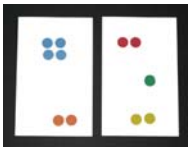
Ten Frame Dot Cards 0—9



Supply List







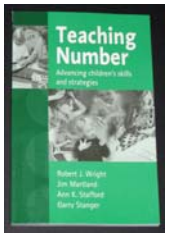








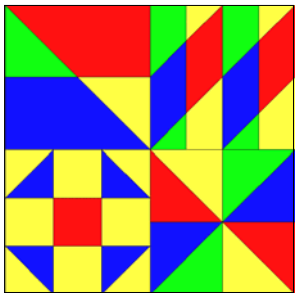




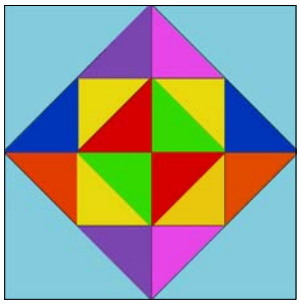
- ✓ **Number track** (1-5; 6-10; 11-15; 16-20; 21-25; 26-30; blank)
- ✓ **Arrow cards**
- ✓ **Number cards** (sets: 1-10; 1-20; 15-25; 1-30; 1-50; 1-100)
- ✓ **Dice** (sets: dots, numbered)
- ✓ **Ten frame cards**
- ✓ **Counters** (two-colored)
- ✓ **Sliding card** (sliding strip 1-100 cut into strips of 1-25, 26-50, 51-75, 76-100)
- ✓ **Bead string**
- ✓ **Rekenrek**
- ✓ **Dot cards**
- ✓ **Playing cards**
- ✓ **Base ten blocks**
- ✓ **100s chart** (class size and individual size—laminated)

✓ **Optional:**

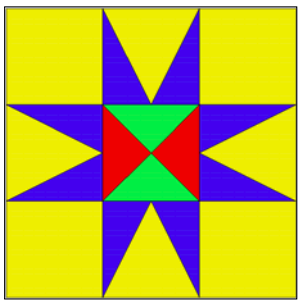
Wright, R. J., Martland, J., Stafford, A. K., & Stanger, G. (2002). *Teaching Number: Advancing children's skills and strategies 2nd Edition.* Thousand Oaks, CA: Sage/Paul Chapman Publications.



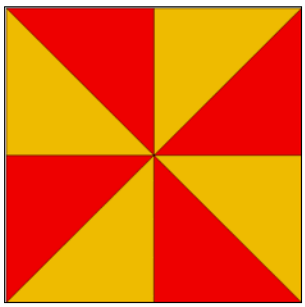
PROBLEM SOLVING



NUMBER WORK



INSPECTING EQUATIONS



FLUENCY & MAINTENANCE

Appendix

Notes: Kindergarten Problem Types

Date:	Join, Result Unknown			Multiplication			Separate, Result Unknown			Measurement Division		
Students:	Direct Modeling	Counting on	Relation-ships	Direct Modeling	Counting on	Relation-ships	Direct Modeling	Counting on	Relation-ships	Direct Modeling	Counting on	Relation-ships

Date:	Join, Result Unknown			Multiplication			Separate, Result Unknown			Measurement Division		
Students:	Direct Modeling	Counting on	Relation-ships	Direct Modeling	Counting on	Relation-ships	Direct Modeling	Counting on	Relation-ships	Direct Modeling	Counting on	Relation-ships

Notes: First Grade Problem Types

Date:	Join, Result Unknown Part, Part, Whole, Whole ?			Multiplication			Separate, Result Unknown			Measurement Division		
Students:	Direct Modeling	Counting on	Relation-ships	Direct Modeling	Counting on	Relation-ships	Direct Modeling	Counting on	Relation-ships	Direct Modeling	Counting on	Relation-ships

Date:	Join, Result Unknown Part, Part, Whole, Whole ?			Multiplication			Separate, Result Unknown			Measurement Division		
Students:	Direct Modeling	Counting on	Relation-ships	Direct Modeling	Counting on	Relation-ships	Direct Modeling	Counting on	Relation-ships	Direct Modeling	Counting on	Relation-ships

Notes: First Grade Problem Types

Date:	Join, Change Unknown			Compare			Separate, Change Unknown			Partitive Division		
Students:	Direct Modeling	Counting on	Relation-ships	Direct Modeling	Counting on	Relation-ships	Direct Modeling	Counting on	Relation-ships	Direct Modeling	Counting on	Relation-ships

Date:	Join, Change Unknown			Compare			Separate, Change Unknown			Partitive Division		
Students:	Direct Modeling	Counting on	Relation-ships	Direct Modeling	Counting on	Relation-ships	Direct Modeling	Counting on	Relation-ships	Direct Modeling	Counting on	Relation-ships

Notes: Second Grade Problem Types

Date:	Join or Separate, Result Unknown			Part, Part, Whole, Part Unknown			Join, Change Unknown			Separate, Change Unknown		
Students:	Direct Modeling	Counting on	Relation-ships	Direct Modeling	Counting on	Relation-ships	Direct Modeling	Counting on	Relation-ships	Direct Modeling	Counting on	Relation-ships

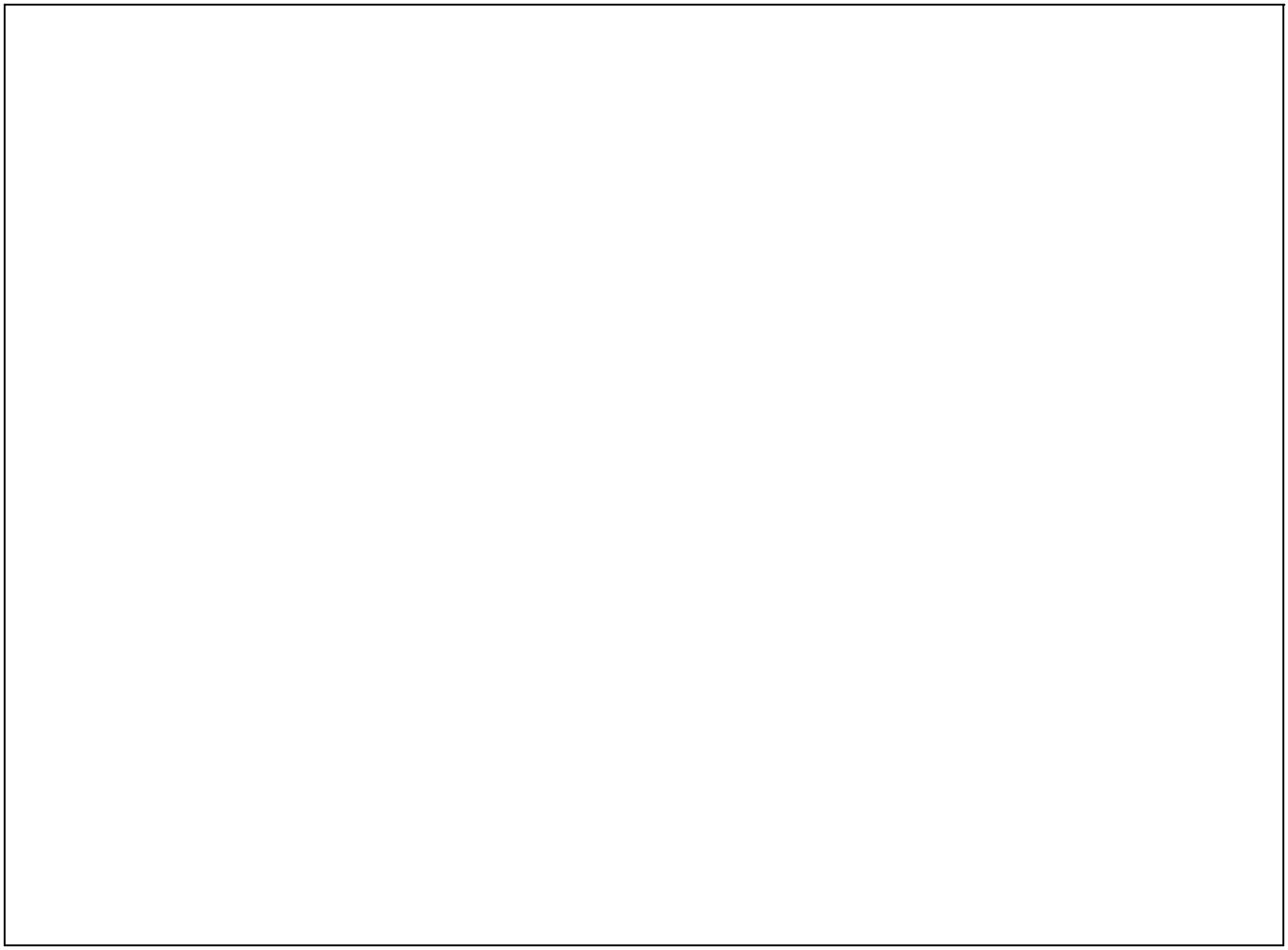
Date:	Join or Separate, Result Unknown			Part, Part, Whole, Part Unknown			Join, Change Unknown			Separate, Change Unknown		
Students:	Direct Modeling	Counting on	Relation-ships	Direct Modeling	Counting on	Relation-ships	Direct Modeling	Counting on	Relation-ships	Direct Modeling	Counting on	Relation-ships

Notes: Second Grade Problem Types

Date:	Compare, Difference Unknown			Multiplication			Measurement Division			Partitive Division		
Students:	Direct Modeling	Counting on	Relation-ships	Direct Modeling	Counting on	Relation-ships	Direct Modeling	Counting on	Relation-ships	Direct Modeling	Counting on	Relation-ships

Date:	Compare, Difference Unknown			Multiplication			Measurement Division			Partitive Division		
Students:	Direct Modeling	Counting on	Relation-ships	Direct Modeling	Counting on	Relation-ships	Direct Modeling	Counting on	Relation-ships	Direct Modeling	Counting on	Relation-ships

Al had _____ stickers. He gave them to three friends. Each friend got the same number of stickers.



How many stickers did each friend get?_____

Name _____ Date _____

Name _____ Date _____

Super Duper Sticker Day

1. Fran made a picture with 12 triangles and 2 squares.

   How many shapes did she have in her picture? _____ _____
   number word

   Show your thinking steps:

2. Nick made a picture with 11 shapes. He took 2 shapes off.

   How many shapes did he have in his picture? _____ _____
   number word

   Show your thinking steps:

3. Alexie made a beautiful picture with 19 squares and 2 triangles.

   How many shapes did Alexie have on her picture? _____ _____
   number word

   Show your thinking steps:

4. Nita took 15 stickers but she put 3 back.

   How many stickers did Nita have left? _____ _____
   number word

   Show your thinking steps:

Name _____ Date _____

Jim picked 6 red flowers and 7 yellow flowers. How many flowers in all did Jim pick? _____ flowers	Stan had 5 big flowers and 9 little flowers. How many flowers did Stan have all together? _____ flowers
I had 8 plants I put 2 plants in each pot. How many pots did I put plants in? _____ pots	Rosalie saw 3 plants. Each plant had 3 flowers on it. How many flowers did Rosalie see? _____ flowers
Jaquan saw 8 ants on a plant. Then he saw 5 ants on another plant. How many ants did Jaquan see all together? _____ ants	My friend had 12 seeds. He put 2 seeds in each pot. How many pots did he put seeds in? _____ pots
I had a pot with 9 flowers in it. I gave away 3 flowers. How many flowers were still in my pot? _____ flowers	Martin had 8 pictures of flowers. He made 6 more pictures. How many pictures of flowers did Martin have in all? _____ pictures
I saw 3 big flowers and 8 little flowers. How many flowers is that all together? _____ flowers	June picked 4 flowers. Sal picked 8 flowers. How many flowers did June and Sal have then? _____ flowers

Fill in the missing numbers:

_____, _____, _____, **31** _____, _____, _____,

Making Shapes with Square Tiles

Grade: Kindergarten

MMSD Standard: Geometry (page 26)

Investigate basic two-dimensional polygons
Child: talks about the results of putting together and taking apart shapes
talks about attributes

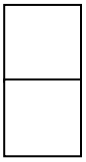
Materials:

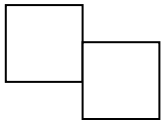
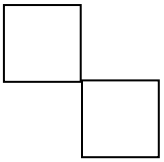
- Set of 1 inch square tiles
- Overhead 1 inch square tiles if using an overhead projector to demonstrate
- Supply of 1 inch graph paper
- Transparency of 1 inch graph paper if using an overhead projector to demonstrate

Time: Initial instruction, 30 minutes
Can become a center or repeated activity throughout the year

Directions:

Demonstrate how to put the squares together to make an interesting shape.
Tiles must share a side from corner to corner:

Like this:  *or this:* 

Not like this:  *or this:* 

Play with the tiles to make a variety of shapes (polygons).

Explain how to make a record of each shape made by coloring the squares on the graph paper.

Engage the class is discussing the records of the shapes they have made. They can talk about:

- number of squares in each shape
- number of color in each shape
- smaller shapes within the new polygon created because of the colors used
- similarity and differences between two shapes
- what they like about the shapes

5 Square Tiles
Pentominoes

Grade: First Grade

MMSD Standard: Geometry (page 26, page 31)

Investigate basic two-dimensional polygons . . . .
Child:

- talks about the results of putting together and taking apart shapes
- sorts shapes according to attributes
- **compares** and talks about attributes

Materials:

Set of five 1 inch square tiles for each student
Overhead 1 inch square tiles if using an overhead projector to demonstrate
Supply of 1 inch graph paper, at least 2 pages per student
1 envelope for each student to hold his/her completed pentominoes

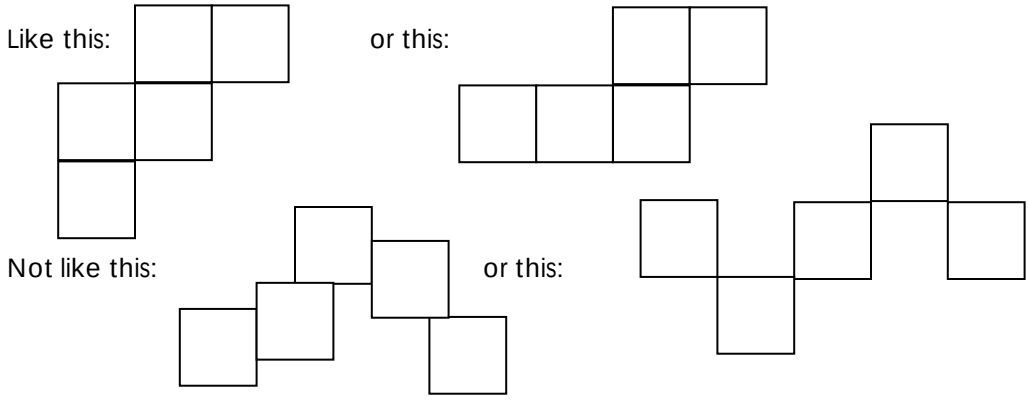
Time: Initial instruction, 45 minutes

Can become a center or a series of activities (30 minutes each)

Directions:

First Activity:

Demonstrate how to put the squares together to make a design.
Tiles must share a side from corner to corner:



Demonstrate how to rearrange the five tiles to make a variety of polygons using all five squares.

Explain that it's each student's job to find as many different ways to put five squares together. They are going to find as many polygons as they can make with 5 squares.

To make a record of each polygon, students will color each arrangement on graph paper.

After about 20 minutes, ask students to stop making new polygons and begin to cut out each of the polygons they have created on the graph paper.

Students should write her/his name on an envelope and put their polygons in the envelope.

Second Activity:

Each student should put out all of the polygons they have made using five 1 inch square tiles.

Direct them to look at their polygons and find any that match.

Facilitate a conversation to determine how many different polygons (shapes) there are.

- You might ask a student to hold up a shape and have the students put that shape back in their envelope.
- Ask if you may keep that polygon to put on a class bulletin board or poster.
- Continue in this manner until all the different shapes the class has made have been collected.
- At some time in the conversation, engage in a discussion about whether shapes that are the flip or rotation of each other are different or not.
- Allow the students to discuss the reasons for thinking as they do.
- Use questions to help them focus their reasoning and eventually come to an agreed upon understanding – flipping or rotating (turning) do not make the shapes different even though they might look different.

Create a bulletin board or poster with the unique shapes identified by the class. You will need to move the shapes in the next activity so do not glue them down on a poster, just lightly tape them.

Third Activity:

Direct the group's attention to the bulletin board or poster displaying the shapes.

Ask: "Have we found all the different ways we can put five squares together to make new polygons?"

"These polygons are made by putting 5 squares together. They are called pentominoes."

Engage the group in a conversation about how they might organize the shapes to decide if they have found all of the possibilities.

Without giving away the number of possibilities, challenge and guide them to think of more ways to put five squares together to create all twelve possibilities.

Have each student make the shapes they are missing so each student has a set of 12 paper pentominoes.

Fourth Activity:

Direct each student to sort their 12 pentominoes by the number of squares in each shape.

After a bit of work, they should realize that there is only one group since every pentominoe is 5 squares big.

Demonstrate how to count the units in the perimeter of each pentominoe. Direct the students to count the units in the perimeters and put the shapes with the same size perimeters together.

Fifth Activity:

Demonstrate how to fold a paper pentominoe along the lines and see if it can be folded to make a box without a top. Show one that does and one that does not.

Direct the students to sort their pentominoes into two groups, ones that do fold to make a box without a top and ones that do not.

After students have sorted their paper pentominoes, create a Venn to communicate the sort.

Students should then make a record of the shapes that fold to make a box without a top by drawing the pentominoes on a sheet of 1 inch square graph paper.

6 Square Tiles
Hexominoes

Grade: Second Grade

MMSD Standard: Geometry (page 27, page 31)

Investigate basic two-dimensional polygons . . . .

Child:

predicts the results of putting together and taking apart shapes
sorts and **classifies** shapes according to attributes
compares and talks about attributes

Materials:

Set of six 1 inch square tiles for each student
Overhead 1 inch square tiles if using an overhead projector to demonstrate
Supply of 1 inch graph paper, at least 2 pages per student
1 envelope for each student to hold his/her completed hexominoes

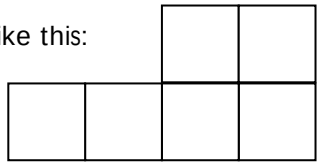
Time: Initial instruction, 45 minutes
Can become a center or a series of activities (30 minutes each)

Directions:

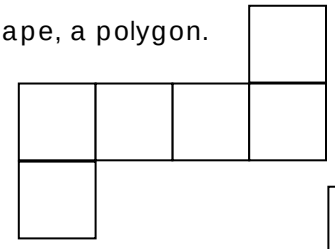
First Activity:

Demonstrate how to put the squares together to make a shape, a polygon.
Tiles must share a side from corner to corner:

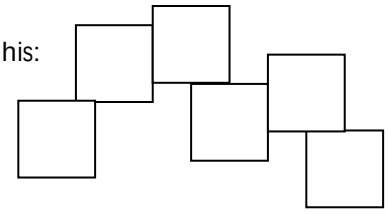
Like this:



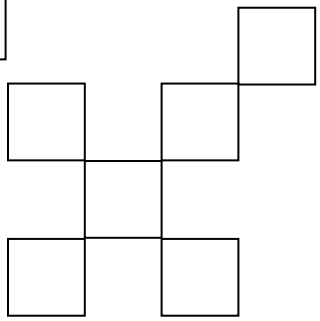
or this:



Not like this:



or this:



Demonstrate how to rearrange the six tiles to make a variety of polygons using all six squares.

Explain that it's each student's job to find as many different ways to put six squares together. They are going to find as many polygons as they can make with 6 squares.

To make a record of each polygon, students will color each arrangement on graph paper.

After about 20 minutes, ask students to stop making new polygons and begin to cut out each of the polygons they have created on the graph paper.

Students should write her/his name on an envelope and put their polygons in the envelope.

Second Activity:

Each student should put out all of the polygons made using six 1 inch square tiles.

Direct them to look at their polygons and find any that match.

Facilitate a conversation to determine how many different polygons (shapes) there are.

- You might ask a student to hold up a shape and have the students put that shape back in their envelope.
- Ask if you may keep that polygon to put on a class bulletin board or poster.
- Continue in this manner until all the different shapes the class has made have been collected.
- At some time in the conversation, engage in a discussion about whether shapes that are the flip or rotation of each other are different or not.
- Allow the students to discuss the reasons for thinking as they do.
- Use questions to help them focus their reasoning and eventually come to an agreed upon understanding – sliding, flipping or rotating (turning) do not make the shapes different even though they might look different.

Create a bulletin board or poster with the different shapes identified by the class. You will need to move the shapes in the next activity so do not glue them down on a poster, just lightly tape them.

Third Activity:

Direct the group's attention to the bulletin board or poster displaying the shapes.

Ask: "Have we found all the different ways we can put six squares together to make new polygons? These polygons are made by putting 6 squares together so they are called hexominoes."

Engage the group in a conversation about how they might organize the shapes to determine that they have found all of the possibilities.

Without giving away the number of possibilities, challenge and guide them to think of more ways to put six squares together to create all 35 possibilities.

Have each student make the shapes they are missing so each student has a set of 35 paper hexominoes.

Fourth Activity:

Direct each student to sort their 35 hexominoes by number of squares in each shape. After a bit of work they should realize that there is only one group since every hexomino is 6 squares big.

Demonstrate how to count the units in the perimeter of each hexomino. Direct the students to count the units in the perimeters and put the shapes with the same size perimeters together.

Fifth Activity:

Demonstrate how to fold a paper hexomino along the lines and see if it can be folded to make a cube.

Show one that does and one that does not fold to make a cube.

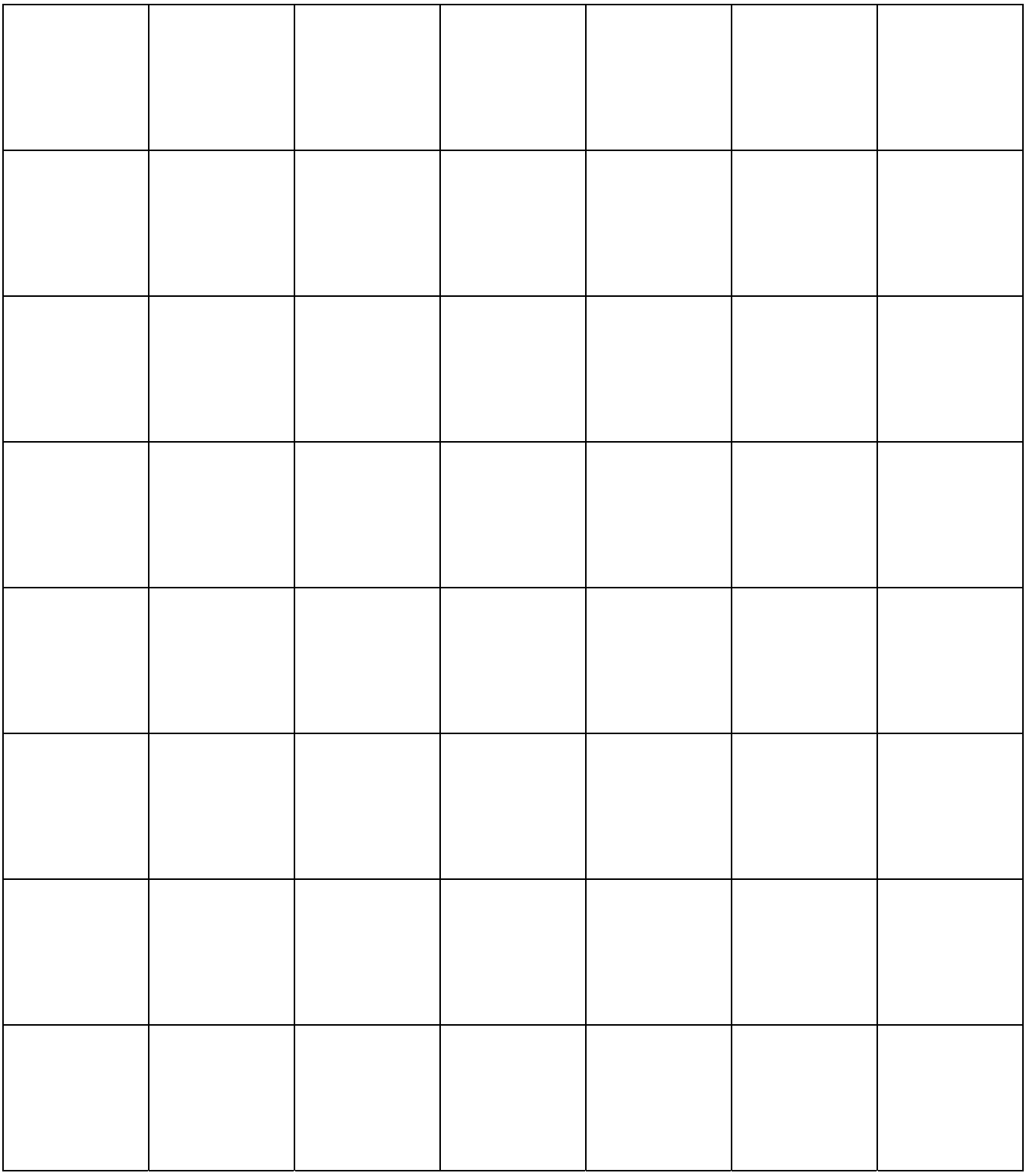
Direct the students to sort their hexominoes into two groups, ones that do fold to make a cube and ones that do not.

After students have sorted their paper hexominoes, create a Venn diagram to communicate the sort.

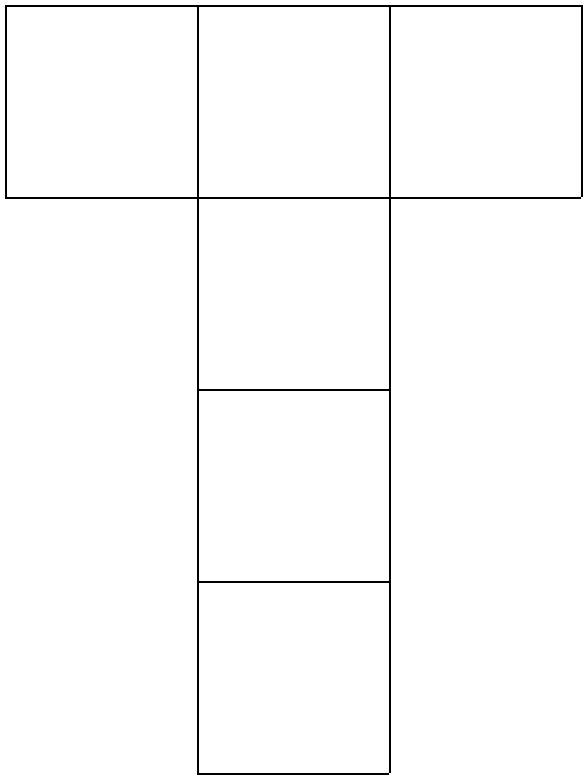
Students should then make a record of the shapes that fold to make a cube. Draw the hexominoes that fold to make a cube on a sheet of 1 inch square graph paper.

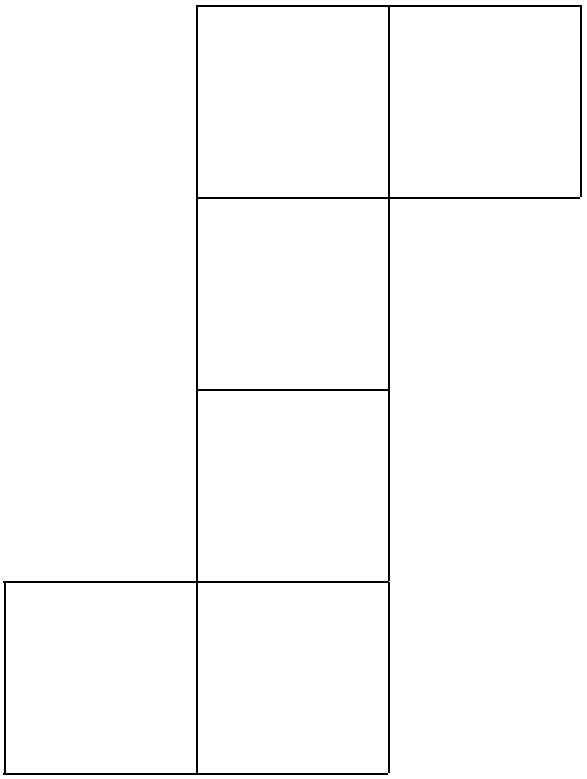
Sixth Activity:

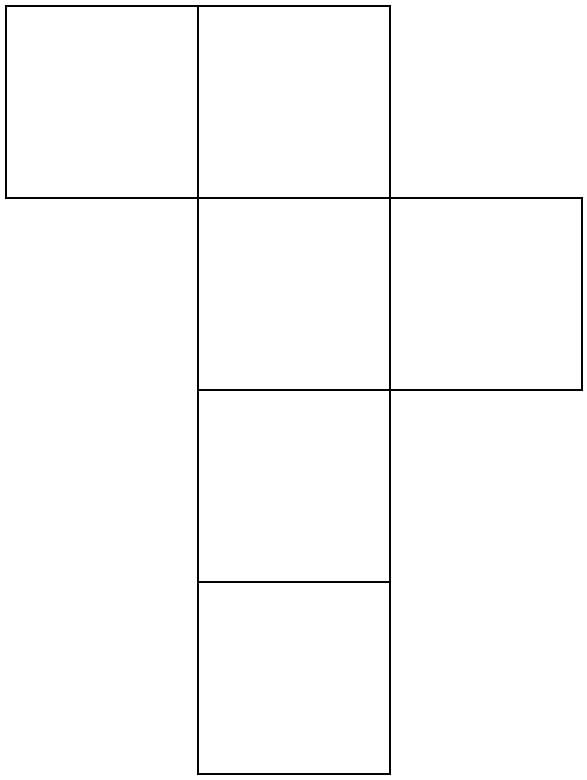
Given a set of nets, each of which can be folded to make a cube, put dots on the squares of the net so that when folded, the cube will look like a die (opposite sides add to seven).

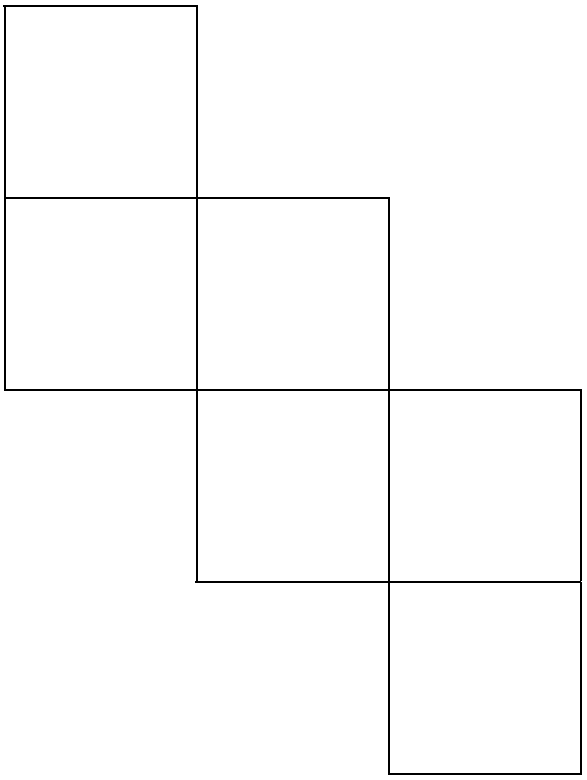


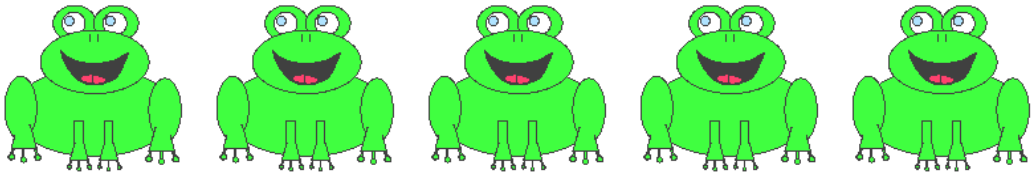
Put dots on the squares of the nets to make a die when folded into a cube (opposite sides add to 7)











Five Green and Speckled Frogs

Traditional Rhyme

Five green and speckled frogs
Sat on a speckled log
Eating some most delicious bugs.
Glub! Glub!
One jumped into the pool
Where it was nice and cool.
Then there were four green speckled frogs.
Glub! Glub!

Four green and speckled frogs
Sat on a speckled log
Eating some most delicious bugs.
Glub! Glub!
One jumped into the pool
Where it was nice and cool.
Then there were three green speckled frogs.
Glub! Glub!
(Continue counting down to zero frogs)

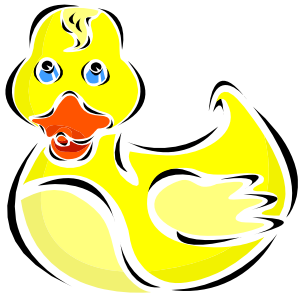
Then there were no more speckled frogs.



Five Little Seashells

Traditional Rhyme

Five little seashells
sleeping on the shore –
SWISH went a big wave;
then there were four.
Four little seashells,
quiet as can be –
SWISH went a big wave;
then there were three.
Three little seashells
pearly and new –
SWISH went a big wave;
then there were two.
Two little seashells
lying in the sun –
SWISH went a big wave;
then there was one.
One little seashell
left all alone –
I put it in my bucket
and then I took it home.



Five Yellow Ducks

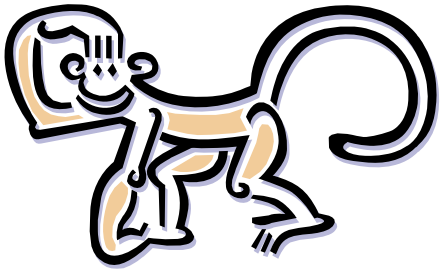
Traditional Rhyme

Five yellow ducks went swimming one day,
Across the pond and far away.
Mother duck said,
"Quack-quack, quack-quack."
And four yellow ducks
came swimming back.

Four yellow ducks went swimming one day,

(Continue until there are zero ducks.)

Grandmother duck went swimming one day,
Across the pond and far away.
Grandmother duck said,
"Quack-quack, quack-quack."
And five yellow ducks came swimming back



Five Little Monkeys

Traditional Rhyme

Five little monkeys jumping on the bed, –
one fell off and bumped his head.
Mother called the doctor,
and the doctor said,
"No more monkeys jumping on the bed!"

Four little monkeys jumping on the bed,
one fell off and bumped his head.
Mother called the doctor,
and the doctor said,
"No more monkeys jumping on the bed!"
(Continue until there are zero monkeys)

Five Little Bunny Rabbits

Traditional Rhyme



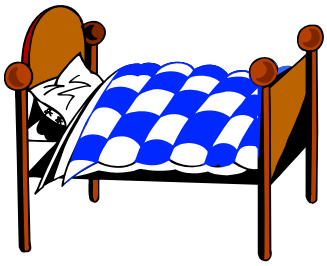
Five little bunny rabbits
Sitting by the door,
One hopped away,
And then there were four.

Chorus:
Hop, hop, hop, hop,
See how they run!
Hop, hop, hop, hop,
They think it's great fun!

Four little bunny rabbits
Under a tree,
One hopped away,
And then there were three.

Chorus:
Three little bunny rabbits
Looking at you,
One hopped away,
And then there were two.

Chorus:
Two little bunny rabbits,
Resting in the sun,
One hopped away,
And then there was one.



Ten in the Bed

Traditional Rhyme

There were ten in the bed,

And the little one said,

"Roll over! Roll over!"

So they all rolled over,

And one fell out.

There were nine in the bed . . .

(Repeat sequence until there is only one in the bed)

There was one in the bed,

And the little one said,

"Goodnight!"



Ten Fat Sausages

Traditional Rhymes

Ten fat sausages sizzle in a pan.

One went pop,

Another went bang!

Eight fat sausages sizzle in a pan.

One went pop,

Another went bang!

Six fat sausages sizzle in a pan. . . .

(Continue counting down by two's until zero)

Then there were no fat sausages

Left in the pan!

The 20 Bead String



Basic Information

Twenty beads are on a string.

There are 5 red, then 5 white, then 5 red, and then 5 white beads on the string.

(If you make your own bead string, use highly contrasting colors if you do not use red and white beads.)

Extra beads are stored on the end of the string that is to the child's right.

The selected beads are at the end of the string that is to the child's left.

Always begin counting with the first set of red beads.

The bead string was developed at the Freudenthal Institute, The Netherlands.

Ways to Use the 20 Bead String

Count Up (Forward)

- Take 1 bead at a time.
- Move the bead from the right to the left.
- Count by one's as each bead is moved to join the set at the left end of the string.

Subitize

- The teacher moves a set of beads toward the left end of the string.
- Show the selected set to the students for a short period of time.
- Ask "How many beads do you see?"
- Repeat this to show sets of 1, 2, 3, 5, and then 10.
- As the students gain skill in identifying the quantity of the set, give them less and less time (less than a second) to view it.

  Always take time to check each response by having a student count the set. Engage the students in a conversation about what they did to help them know how many beads were in the set. When children show success in identifying sets of 1, 2, 3, 5, and 10, move to showing sets of 4, or 6, 9, 7 and asking the students to identify the quantities. The conceptual understanding needed to use 0, 5, 10, 15 and 20 as landmarks develops over time with repeated experiences.

Show Sets

- Ask a child, "Can you show 2 in one push?"
- Without counting, the child should separate two beads from the storage area at the right end of the string.
- The child then moves the set of 2 (together) from the right to the left end of the string.
- As children show success in doing this, ask them to use one push to show sets of 3, 5, and 10. Then move to showing sets of 4, 6, 9, 7 and finally 8! Continue on using numbers up to 20.

Matching Numerals and Sets

- The teacher shows a set of beads.
- Ask the children to hold up the card that tells how many beads are shown.
- If children have their own bead string, the teacher can hold up a numeral card and ask the students to show that many beads with one push.

Counting On

- The teacher or a child uses one push to create a set at the left side of the string.
- The teacher wonders, "If we move (or another verb that the children understand) 1 bead to this set (group) of beads, how many beads will we have then?"
- The child says the number of the selected group at the left of the string, takes a bead from the storage at the right, moves it to the group at the left and says the new number.

Part, Part, Whole Relationships (Sums less than or equal to 10)

- The teacher or a child uses one push to create each addend.
- The contrasting colors of the beads provide the context for seeing the sum in relationship to either five or ten. For instance, a child pushes 6 in one push and then 3 in one push.
- In the set of beads that is the total, a child sees five red and five white beads.
- Knowing that 10 is five red beads and 5 white beads, this newly formed total is 1 less than 10, 9.

Showing Doubles

- A teacher can use the bead string to show doubles. Change the storage area by moving ten beads to each end of the string. The middle of the string becomes the workspace.
- Ask the child: "Push one (or two or more) beads from each end to meet in the middle."
- The colors will be contrasting so the double is easily visible. For instance, two from the left (white beads) and two from the right (red beads) meet in the middle of the string to show the double 2 + 2.

Across Ten Facts

- The 20 bead string gives a strong visual representation of decomposing the second addend to make ten and then add the remaining quantity to 10.
- To do this, a teacher or a child pushes the beads to model the first addend. Then a teacher or a child pushes the beads to represent the second addend.
- There can be a little space between the two groups of beads.
- Even with a small space between the groups, a child can easily see that there are two complete groups of five by looking at the groups of contrasting colors.
- Knowing that two groups of five is ten beads, a child just needs a quick look at the remaining beads to determine the quantity and add that amount to 10.

Based on Andrews, A. G. (2005). *Using the 20 bead string.* Unpublished manuscript.

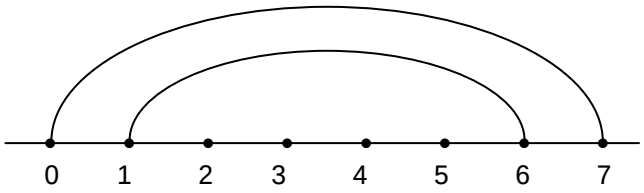
Rainbow Numbers
A Number Sense Idea

Rainbow numbers help students recall part, part, whole relationships and begin to build fluency in decomposing numbers. Seven seems to be a good starting number. Prior to starting this activity, students have been doing lots of part, part, whole story problems for sums to 10 and beyond.

Begin by drawing an empty number line and placing numbers on it from 0 to 7.

Ask: "What number can you put with 7 to make a total of 7?" Students will respond 0.
Draw an arch from 7 to 0.

Continue by asking "What number can you put with 6 to make a total of 7?
Draw an arch from 6 to 1.



Continue to count down the number line. Make sure to ask "What number can you put with 3 to make 7" and trace over the arch, going from 3 to 4.

As the students identify each part-part whole relationship, draw or trace the arch to connect the numbers. You may want to use colored chalk or markers!

When all the arches are drawn, make an organized list of the pairs of addends:

We connected 7 with 0	7 + 0
We connected 6 with 1	6 + 1
And so on . . .	5 + 2
	4 + 3
	3 + 4
	2 + 5
	1 + 6
	0 + 7

Within a few days, repeat the process for the number 5. Then, a day or two later, repeat the process finding all the pairs of addends to make the sum of 8.

If students are anticipating the process, it's appropriate to see if some students can create the organized list before creating the number line and drawing arches. They can create the rainbow to check and see if they have listed all the pairs.

Name _____ Date _____

Things In This Counting Book

Title _____

_____	five	_____	_____

Directions: Find the number word in the book. Draw a picture of one of the things that number counted. Write the number words that come before and after this number word. Find the things for those number words. Draw one of the things that each number counted.

Name _____ Date _____

Things You Can Count in This Counting Book

Title_____

The biggest number in the book is _____.

The smallest number in the book is _____.

The best thing to count is

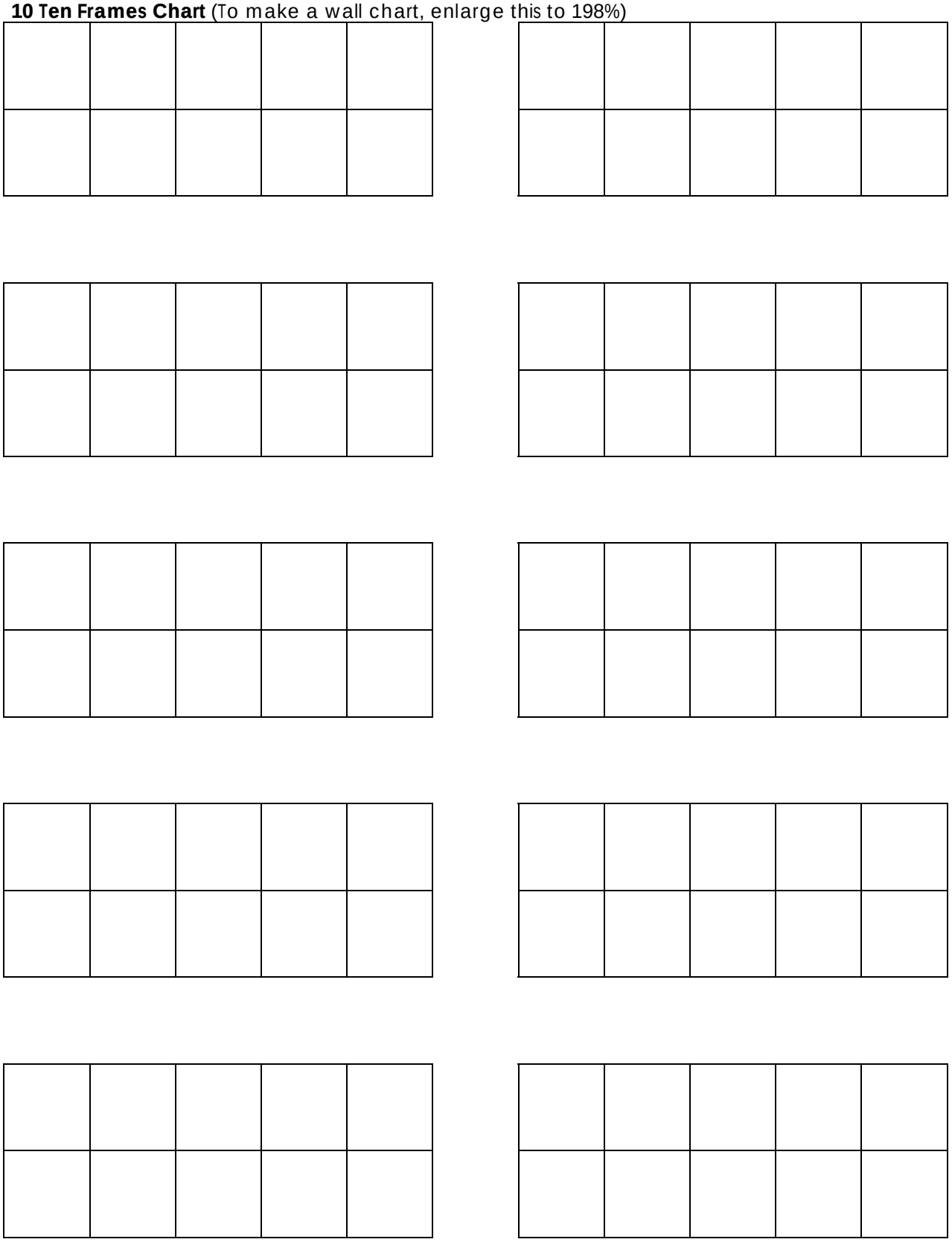
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You can also count

_____

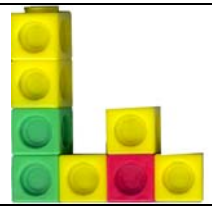
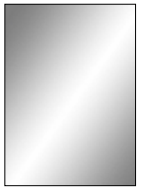
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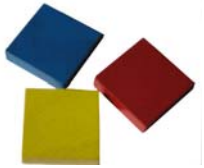
10 Ten Frames Chart (To make a wall chart, enlarge this to 198%)



Number and Operations	
	two-color counters
	buttons
	Unifix cubes
	place value blocks

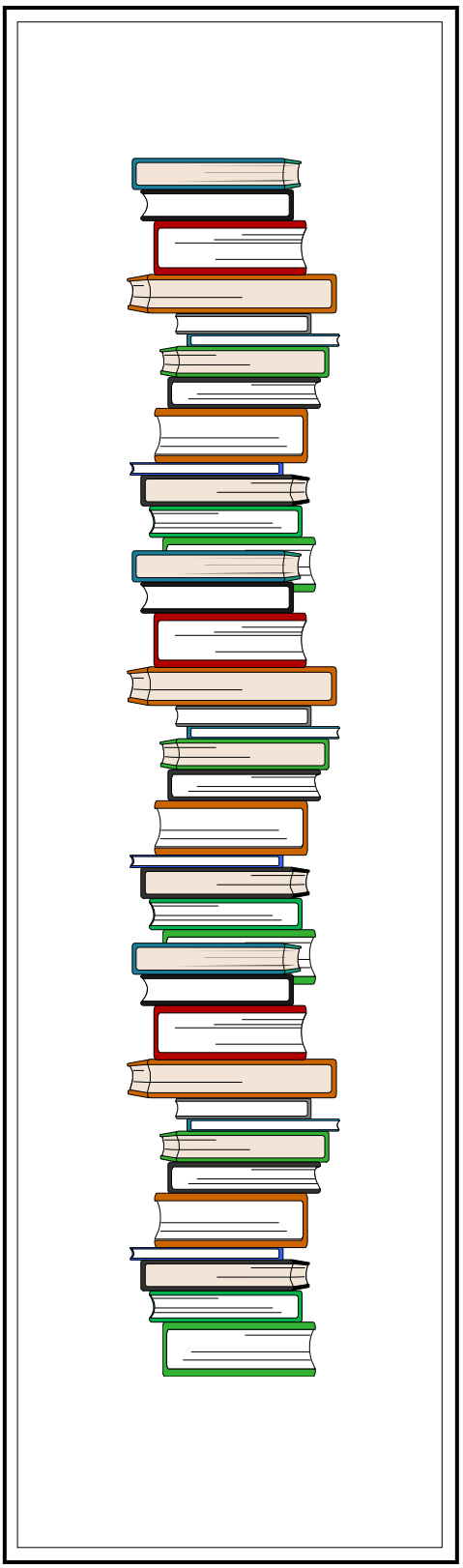
	counters
	calculators
Geometry	
	geometric solids
	Geoblocks

	**Multi-links**
	**Polydrons**
	**mirrors**
	**geoboards**

	**square tiles**
	**pattern blocks**
	**pentominoes**
	**wooden cubes**

	**tangrams**
Measurement	
	**balances**
	**rulers**

	**Attribute blocks**
	**measuring tools**
	**measuring tapes**
	**links**



Professional Resources

Andrews, A. G. & Trafton, P. (2002). *Little kids – powerful problem solvers: Math stories from a kindergarten classroom*. Portsmouth, NH: Heinemann.

Andrews, A. G. (2005). *Using the 20 bead string.* Unpublished manuscript.

Bachman, V. (2003). *First-grade math: A month-to-month guide*. Sausalito, CA: Math Solutions Publications.

Brizuela, B. M. (2004). *Mathematical development in young students.* New York, NY: Teachers College Press.

Carpenter, T. P., Fennema, E., Loef Franke, M., Levi, L., & Empson, S. B. (1999). *Children's mathematics: Cognitively guided instruction.* Portsmouth, NH: Heinemann.

Carpenter, T. P., Franke, M. L. & Levi, L. (2003). *Thinking mathematically: Integrating arithmetic and algebra in elementary school*. Portsmouth, NH: Heinemann.

Chappell, M.F., Schielack, J. F. & Zagorski, S. (Eds.). (2004). *Empowering the beginning teacher of mathematics: Elementary school*. Reston, VA: National Council of Teachers of Mathematics.

Clarke, B. & Clarke, D. (2004). Mathematics teaching in grades K-2: Painting a picture of challenging, supportive, and effective classrooms. In Rubenstein, R. N. & Bright, G. W. (Eds.). *Perspectives on the Teaching of Mathematics Sixty-sixth Yearbook*. (pp. 67-81). Reston, VA: National Council of Teachers of Mathematics.

Clements, D. H. & Sarama, J. (Eds.). *(*2004). *Engaging young students in mathematics: Standards for early childhood mathematics education.* Malwah, NJ: Lawrence Erlbaum Associates, Publishers.

Confer, C. (2002). *Math by all means: Geometry grades 1-2*. Sausalito, CA: Math Solutions Publications.

Confer, C. (2005). *Teaching number sense: Kindergarten*. Sausalito, CA: Math Solutions Publications.

Confer, C. (2005). *Teaching number sense: Grade 1*. Sausalito, CA: Math Solutions Publications.

Dacey, L., Cavanagh, M., Findell, C. R., Greenes, C. E., Sheffield, L. J. & Small, M. (2003). *Navigating through measurement In prekindergarten – grade 2.* Reston, VA: National Council of Teachers of Mathematics.

Dacey, L.S., & Eston, R. (1999). *Growing mathematical ideas in kindergarten.* Sausalito, CA: Math Solutions Publications.

Dana, M. *Geometry: Experimenting with shapes and spatial relationships-grade 1*. Grand Rapids, MI: Instructional Fair, Inc. (Out of print, see an elementary math resource teacher)

Dana, M. *Geometry: Experimenting with shapes and spatial relationships* - grade 2. Grand Rapids, MI: Instructional Fair, Inc. (Out of print, see an elementary math resource teacher)

Dana, M. *Inside-out math problems: Investigate number relationships & operations.* Grand Rapids, MI: Instructional Fair, Inc. (Out of print, see an elementary math resource teacher)

Dana, M. (1998). *Math Practice Games.* Grand Rapids, MI: Instructional Fair, Inc. (Blackline resource for Grades 1, 2, 3, 4, and 5 available from a math resource teacher)

Dixon-Krauss, Lisbeth. (1996). *Vygotsky in the classroom: Mediated literacy instruction and assessment*. Boston, MA: Allyn and Bacon.

Findell, C. R., Small, M., Cavanagh, M., Dacey, L., Greenes, C. R. & Sheffied, L. J. (2001). *Navigating through geometry in prekindergarten – grade 2.* Reston, VA: National Council of Teachers of Mathematics.

Fuson, K. C., Kalchman, M., & Bransford, J. D. (2005). Mathematical understanding: An introduction. In Donovan, M. S., and Bransford, J. D. (Eds.). *How Students Learn Mathematics In the Classroom.* (pp. 215-256). Washington, DC: The National Academies Press.

Glanfield, F., Bush, William S. & Stenmark, J. K. (Eds.). (2003). *Mathematics assessment: A practical handbook K-2*. Reston, VA: National Council of Teachers of Mathematics.

Greenes, C., Cavanagh, M., Dacey, L., Findell, C., & Small, M. (2001). *Navigating through algebra in prekindergarten through grade 2.* Reston, VA: National Council of Teachers of Mathematics.

Greenes, C. E., Dacey, L., Cavanagh, M., Findell, C. R., Sheffield, L. J., & Small, M. (2003). *Navigating through problem solving and reasoning in kindergarten.* Reston, VA: National Council of Teachers of Mathematics.

Greenes, C. E., Dacey, L., Cavanagh, M., Findell, C. R., Sheffield, L. J., & Small, M. (2004). *Navigating through problem solving and reasoning in grade 1.* Reston, VA: National Council of Teachers of Mathematics.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K.C., Wearne, D. & Hanlie, M. (1997). *Making sense: Teaching and learning mathematics with understanding.* Portsmouth, NH: Heinemann.

Hope, J. A., Leutzinger, L, Reys, B. J. & Reys, R. E. (1988). *Mental math in the primary grades*. Parsippany, NJ: Pearson Learning Group.

Kamii, C. (2004). *Young students continue to reinvent arithmetic 2nd edition*. New York, NY: Teachers College Press.

Kilpatrick, J., Martin, G. W. & Schifter, D. (Eds.). (2003). *A research companion to principles and standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Kilpatrick, J., Swafford, J., & Findell, C. R., (Eds.). (2001). *Adding it up*. Washington, DC: National Academy Press,

Kilpatrick, J., Swafford, J., & Findell. (Eds.). (2002). *Helping students learn mathematics.* Washington, DC: National Academy Press.

Litton, N. (2003). *Second-grade math: A month-to-month guide*. Sausalito, CA: Math Solutions Publications.

Ma, L. (1999). *Knowing and teaching elementary mathematics.* Mahwah, NJ: Lawrence Erlbaum Associates, Publishers.

Madison Metropolitan School District. (2004*). MMSD mathematics grade level K-5 standards*. Madison, WI: Retrieved March 16, 2006 from http://www.madison.k12.wi.us/tnl/standards/math/

Madison Metropolitan School District. (2004). Strategic Plan. Madison, WI.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: An on-line version retrieved March 16, 2006 from http://standards.nctm.org/

Malloy, C. E. (2004). Equity in mathematics education is about access. In Rubenstein, R. N. & Bright, G. W. (Eds.). *Perspectives on the Teaching of Mathematics Sixty-sixth Yearbook*. (pp. 1-14). Reston, VA: National Council of Teachers of Mathematics.

Pilar, R. (2004). Spanish-English language issues in the mathematics classroom. In Ortiz-Franco, L., Hernandez, N. G., & De La Crus, Y. (Eds.). *Changing the faces of mathematics perspectives on Latinos.* (pp. 23-33). Reston, VA: National Council of Teachers of Mathematics.

Richardson, K. (1999). *Developing number concepts: Counting, comparing, and pattern.* Parsippany, NJ: Pearson Education, Inc.

Richardson, K. (1999*). Developing number concepts: Addition and subtraction.* Parsippany, NJ: Pearson Education, Inc.

Richardson, K. (1999). *Developing number concepts: Place value, multiplication, and division.* Parsippany, NJ: Pearson Education, Inc.

Richardson, K. (2002). *Math time: The learning environment*. Peabody, MA: Educational Enrichment, Inc. Didax Educational Resources.

Sawder, J., & Schapelle, B. (Eds.). (2002). *Lessons learned from research.* Reston, VA: National Council of Teachers of Mathematics.

Sharton, S. (2005). *Teaching number sense: Grade 2*. Sausalito, CA: Math Solutions Publications.

Sheffield, Stephanie. (2001). *Lessons for first grade*. Sausalito, CA: Math Solutions Publications.

Sheffield, Stephanie. (2002). *Lessons for introducing place value.* Sausalito, CA: Math Solutions Publications.

Small, M., Sheffield, L. J., Cavanagh, M., Dacey, L., Findell, C. R. & Greenes, C. E. (2004). *Navigating through problem solving and reasoning in grade 2.* Reston, VA: National Council of Teachers of Mathematics.

Sutton, J. & Krueger, A. (Eds.). (2002). *EDThoughts: What we know about mathematics teaching and learning*. Aurora, CO: Mid-continent Research for Education and Learning.

Trafton, P. R., & Thiessen, D. (1999). *Learning through problems: Number sense and computational strategies.* Portsmouth, NH: Heinemann.

Van de Walle, J. A. (2001). *Elementary and middle school mathematics*. New York, NY: Longman.

Van de Walle, J. A. & Lovin, L. H. (2006). *Teaching student-centered mathematics: Grades K-3.* Boston, MA: Allyn and Bacon Pearson Education, Inc.

von Rotz, L., & Burns, M. (2002). *Lessons for algebraic thinking.* Sausalito, CA: Math Solutions Publications.

Wheatley, G. H. & Reynolds, A. M. (1999). *Coming to know number: A mathematics activity resource for elementary school teachers.* Tallahassee, FL: Mathematics Learning.

White, D. Y. (2000). Reaching all students mathematically through questioning. In Strutchens, M. E., Johnson, M. L. & Tate, W. F. (Eds.). *Changing the face of mathematics: Perspectives on African-Americans.* (pp. 21-32). Reston, VA: National Council of Teachers of Mathematics.

Wisconsin Department of Public Instruction. *Wisconsin model academic standards.* Madison, WI*:* Retrieved March 16, 2006 from http://dpi.wi.gov/standards/matintro.html

Wisconsin Department of Public Instruction. (2005). *Wisconsin knowledge and concepts examination assessment framework.* Madison, WI*:* Retrieved March 16, 2006 from http://www.dpi.state.wi.us/oea/doc/wkce_math_framework05.doc

Wright, R. J., Martland, J., Stafford, A. K., & Stanger, G. (2006). *Teaching number advancing children's skills & strategies 2nd edition*. Thousand Oaks, CA: Sage Publications Inc.